This course supports the assessment for RMT2. The course covers 2 competencies and represents 2 competency units.

**Introduction**

**Overview**
This is the culminating assessment in the mathematics portion of your program, and as such, it is quite different than any of the other assessments in the program. There are no new learning resources. Instead, you will apply knowledge or discuss connections among already-learned topics. You will not be given as much detailed and specific guidance on exactly where to find relevant topics within your existing learning resources. The nature of a culminating course demands a greater degree of independence and sophistication from the student. By applying and connecting your mathematical knowledge, you will deepen your knowledge and therefore develop yourself as a teacher.

Please note that unlike prior courses, this one will refer you back to past courses in order to review the activities found there in as needed (there is actually no new material to be found in this course).

This course provides guidance to help you demonstrate the following 2 competencies:

- **Competency 210.5.1: Mathematical Modeling**
  The graduate uses mathematics concepts to represent and understand quantitative relationships, investigate patterns, make generalizations, formulate mathematical models, make predictions, and validate results.

- **Competency 210.5.2: Mathematical Connections**
  The graduate understands and can explain the wide applicability of linear and abstract algebra to other areas of mathematics, including the calculus and geometry.

**Teaching Dispositions Statement**
Please review the [WGU Statement of Teaching Dispositions](#).

**Course Mentor Assistance**
As you prepare to successfully demonstrate competency in this subject, remember that course mentors stand ready to help you reach your educational goals. As subject matter experts, mentors enjoy and take pride in helping students become reflective learners, problem solvers, and critical thinkers. Course mentors are excited to hear from you and eager to work with you.

Successful students report that working with a course mentor is the key to their success. Course mentors are able to share tips on approaches, tools, and skills that can help you apply the content you're studying. They also provide guidance in assessment preparation strategies and troubleshoot areas of deficiency. Even if things don’t work out on your first try, course mentors act as a support system to guide you through the revision process. You should expect to work with course mentors for the duration of your coursework, so you are welcome to contact them as soon as you begin. Course mentors are fully committed to your success!
Preparing for Success

The information in this section is provided to detail the resources available for you to use as you complete this course.

Learning Resources
This course will rely on learning resources from past courses. You should be prepared to access past courses.

Pacing Guide
The pacing guide suggests a weekly structure to pace your completion of learning activities. It is provided as a suggestion and does not represent a mandatory schedule. Follow the pacing guide carefully to complete the course in the suggested timeframe.

- Pacing Guide: Mathematical Modeling and Connections

Note: This pacing guide does not replace the course. Please continue to refer to the course for a comprehensive list of the resources and activities.

Mathematical Modeling and Connections Task 1: Population Modeling

The first task is about linear and exponential models for world population growth. Some estimate that the world population will almost double from the current level within 50 years. What sort of consequences would that have? Can the world sustain this growth? Can a country have a negative growth rate? What might cause a negative growth rate?

Population Modeling
How can mathematics, especially statistics, be used to model the growth of populations? You are very familiar with a variety of functions, including linear, exponential, and logarithmic, to name just three. Consider the following questions as you complete this section:

- When you think of population growth, which function jumps to mind?
- Why might a particular function be appropriate or inappropriate?
- How can you test the accuracy of any given mathematical model for population growth?
- What statistical processes might you employ?

This topic addresses the following competencies:

- Competency 210.5.1: Mathematical Modeling
  The graduate uses mathematics concepts to represent and understand quantitative relationships, investigate patterns, make generalizations, formulate mathematical models, make predictions, and validate results.

- Competency 210.5.2: Mathematical Connections
  The graduate understands and can explain the wide applicability of linear and abstract algebra to other areas of mathematics, including the calculus and geometry.

Task 1 preparation videos
Task 1 Guide

Use the Assessment tab and the Preview button or log into TaskStream to read the task. It is about using technology to make predictions. It requires knowledge of Statistics II and technology.

Part A, A1, B, B1, C: Given 11 data points (equally spaced), use technology to develop a linear model, predict the 12th data point, develop an exponential model, predict the 12th data point, and compare both to the actual 12th data point.

Part C1: Use the model that matches the 12th data point most closely to predict the 13th, 21st, and 31st data points.

Part C2: Critique that model's ability to predict those data points.

Carefully use significant digits throughout.

One of the most common causes of failure is misunderstanding significant digits; please read up on them on Wikipedia or PurpleMath if you feel uncertain. One of the other common problems is not knowing how to get Excel to display enough digits to be able to read 8 significant figures. When you’ve got the equation displayed on the trendline graph, right-click the equation. Change the General number format to Number format -- 2nd option instead of 1st. Then increase the Decimal Places from the default of 2 to a LOT more so that you’ll have enough displayed to see the first 9 nonzero digits so the 9th can round the 8th and you can keep 8 significant figures. When you’re reporting final results for population projections (2005 twice, 2010, 2050, and 2100) you’re supposed to shrink down to 4 significant digits to increase readability. That means you need to perform calculations via technology or by hand while you keep 8 or more significant figures, and you may only round to 4 significant figures when the calculations are done and you are reporting final population results for a particular year.

Another common cause of failure is not manipulating the output of the technology enough to match the requirements of the task. For example, the default output mode of Excel will not fulfill the significant digit requirements of the task. Doing this task also requires knowledge of technology to compute the answers. If you feel you need a lot more training on using Excel for trendlines, we highly recommend using Lynda.com, a tech-training website WGU has purchased access to on your behalf. Go to the Library through the Resources tab in your Degree Portal and then when you are on the Library home page click on “Lynda.com Video Library” in the left margin. Search for “Excel trendline” and a few courses should pop up -- open them and look for “trendline” in the section header; there should be one in each course. Linear regression can be done by hand, but that is NOT EXPECTED of you (and exponential regression almost always needs technology). At this stage in your program, it is assumed you are competent with the methods of Statistics and the technological tools to solve
problems, so *showing the algebraic and arithmetic computations is NOT the point*. However, whenever you use a technology, you should give something fairly close to a “click-by-click” narrative of what you did -- write enough detail that anyone with the same knowledge of math and perhaps slightly less knowledge of technology could follow along and get the same result as you. Assume the reader is familiar with the technology, but needs to be told what to do -- they know how to use the technology in general, but they haven't done this process before in specific. If you don't know basics like how to get Excel to give trendlines with the equations visible, you'll either need to do some experimentation in Excel, try typing something relevant into Google or online Office Help such as: trendline format Excel, or pursue Lynda.com as noted above.

*Don't over-think the goals of task 1 – it's just a prompt to get you to run your technology to make two kinds regression and and make some predictions with them.*

We recommend that you use Excel because results and diagrams are very easily pasted into your task and this spreadsheet is very likely available to most students you'll teach.

If you know how to create a linear trendline, you're done with part A.

To finish part B, you can modify the line into an exponential by left-clicking the line and then right-clicking to modify it -- [*the Microsoft Knowledge Base has step-by-step directions for doing this very thing*](https://support.office.com/en-us/article/trendline-format-excel-580560eb-c22f-4f10-93b6-b7c7e43bc53e).

To begin part C, simply use the model that is more accurate in 2005 to make three more predictions (2010, 2050, and 2100).

In part C1, you should critique the model you had to use: Is population growth usually linear? Should the R2 values of the trendlines be considered? What about non-mathematical considerations like the cumulative effective over longer and longer times of natural disasters, advances in agricultural technology, political upheaval or tranquility, advances in medical science, and so on? This part of the task is more about common sense and social science rather than being strictly mathematical. Explain, with examples, what influences you to trust long range predictions less than short range predictions, and that will suffice as a discussion of specific factors that could affect future population values.

**Task 1 Performance Task**

Complete the following task in [*TaskStream*](https://example.com):

- Math Modeling &Connections: Task 1

For details about this performance assessment, see the "Assessment" tab in this course.

**Mathematical Modeling and Connections Task 2:**

**Classroom Recursion**
In this task you will discuss how to introduce a recursion application appropriate for a high school classroom. This is a topic that sometimes gives students trouble, as it is a way to define functions with which they may not yet be familiar. Given that this is the case, what might you do in order to make these mathematical objects less threatening or more familiar?

**Classroom Recursion**

Consider the following questions as you complete this section:

- What applications of recursion would be appropriate to help motivate high school students and to provide the appropriate level of algebraic challenge?
- Are there real-world examples that can be brought to light?
- When are these sorts of mathematical structures used?
- Why are they used?

This topic addresses the following competencies:

- **Competency 210.5.1: Mathematical Modeling**
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**Task 2 Preparation Video**

- **RECURSIVE SEQUENCES** (6 min)

**Task 2 Preparation Readings: summaries of sample submissions**

- Fibonacci
- Purchasing Matchbox Cars Over Time
- Annual interest rate

**Task 2 Guide**

Use the Assessment tab and the Preview button or log into TaskStream to read the task. It has you explain how to teach a recursive function application to a high school class. Requires knowledge of iterative functions and their formulas.

Parts A & B: As if for a teacher colleague, explain how they should teach recursive functions in a high school class. Make sure to cover an the introductory topics and some student-focused examples.

The best frame of mind to take when writing this task is that your teacher colleague is a great guy, but he mostly loves his coaching job and he’s been teaching the lower-level math courses at your school for decades. He’s stepping in for a maternity leave next year and he has to cover a course he hasn’t taught and he’s forgotten most of his own advanced mathematics coursework. Your job this summer is a crash-course to prepare him to teach that course, and
this task is one part of that training -- one that you have to provide him with all the written materials in advance because you’re going on vacation, but he can’t afford to fall behind..

The algebra isn’t that sophisticated, but recursion is a somewhat challenging idea if you haven’t had much exposure to it. Keep things simple -- don’t over-complexify the lesson; for example, it isn’t necessary to get deep into recursion theory and start bumping into proofs by induction and so on. It is enough to provide a nice introduction to iterative formula -- the “next-now” notation and ideas. It is okay to keep it to a savings account with compound interest or something similarly simple. Although having the students use Excel to “drag the formula down” is a great idea, it is probably NOT a good idea to have them use the instant financial functions to figure out problems. Recursive functions ‘next-now’ statements; they are not ‘if-then’ statements like regular functions, where if you know the x, you can compute the y immediately. For example, \( y=2x+9 \) is an ‘if-then’ statement, because if you know the x, you can compute the y. That’s an old-fashioned explicit function. The trick to thinking recursively is to think about how to transform the current value, the “now” in order to define the next one. The same function above ‘starts’ at 9, and every step of ‘increasing one’ adds 2 more to that start. So in ‘next-now’ style, the function is:

\[
\begin{align*}
a_0 &= 9 \\
an+1 &= an+2
\end{align*}
\]

Your compound interest type of problem can also be very simply expressed with recursive notation. Annuities, where you combine monthly deposits with monthly interest on what was already there, have a rather simple recursive representation, and are easily put into Excel or calculator spreadsheets so the number of months to get to a particular goal can be observed on an extended table -- that’s the ‘drag down the formula’ approach. If you need a more concrete example of iteration to help you create one appropriate for students, examine the Trout Pond problem from NCTM Illuminations. Since the task asks you to connect with state and national standards, you ought to do some reading of the new Common Core State Standards for Mathematics. Modeling and Functions in High School are recommended as a starting place. A reasonable alternative to the Common Core would be the Standards for Grades 9-12 set by The National Council of Teachers of Mathematics or your state’s standards if they have not adopted the Common Core.

**Task 2 Performance Task**

Submit the following task in TaskStream:

- Math Modeling & Connections: Task 2

For details about this performance assessment, see the "Assessment" tab in this course.

**Mathematical Modeling and Connections Task 3: Gradient**

You will write about how the gradient is an example of the connection between linear algebra and calculus. Does this connection seem intuitively obvious to you? Why or why not?

**Gradient**

Consider the following questions as you complete this section:
How does the gradient provide an example of the way linear algebra and multivariable calculus can be interwoven?
How might you make these concepts understandable?
What concepts must you fully understand as prerequisites in order to be able to make this connection between these two important branches of mathematics?

This topic addresses the following competencies:

**Competency 210.5.1: Mathematical Modeling**
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**Gradient From Calculus**

Review the relevant topics from the following course

- Calculus III & Analysis

Revisit your prior notebook entries and reflect on how this knowledge can be applied now.

**Dot Product From Linear Algebra**

Read the following section from *Lay's Linear Algebra*, and complete the assigned exercises:

- section 6.1 ("Inner Product, Length, and Orthogonality"), exercises 1–11 odd

Also complete the exercises at the following site:

- [Dot Product (aka Scalar Product) in 2 Dimensions](#)

Reflect on how this knowledge can be applied.

**Task 3 Preparation Videos**

- Knowledge: [THE GRADIENT VECTOR: NOTATION AND DEFINITION](#) (3 min)
- Knowledge: [FINDING THE DIRECTIONAL DERIVATIVE](#) (7 min)
- Knowledge: [VECTORS: THE DOT PRODUCT](#) (8 min)

**Task 3 Preparation Reading**

- [Directional Derivative Examples](#)

**Task 3 Guide**

Use the Assessment tab and the Preview button or log into TaskStream to read the task. It is a general essay about relationships between linear algebra and calculus as well as a specific proof about the directional derivative and the gradient vector. Requires knowledge of
Multivariable Calculus and Linear Algebra.

One thing to start with is how multivariable calculus and linear algebra were probably the first courses where functions of more than one variable were discussed. And it probably can’t be a coincidence that you covered a whole chapter in Calculus III called “Vectors.” The details of the rest of the discussion about connections between the courses is up to you, but a good starting point would be a firm understanding of what each course is about at the fundamental/conceptual level, and for this, a good reference is A Gentle Introduction to the Mathematics Subject Classification Scheme. Wikipedia and other encyclopedia-style references will add depth to that discussion for you.

Part B: Write a proof about gradient vectors and directional derivatives.

It is NOT enough to provide a general discussion of the situation and a few examples where it can be shown to be true. You must go into enough detail that you’re doing and explaining every algebraic and conceptual step from the given situation to the desired conclusion. Whether that is organized into two columns or presented as flowing text or written line-by-line doesn't concern us very much. Just make sure every step is shown and clearly explained -- that is the essence of a good proof.

If you’re recollection of multivariable calculus is fading, please watch a student-recommended video that will help you conceptually and visually understand directional derivatives and gradients.

Please note the definition of directional derivative is a GIVEN in this proof. And the definition of a dot product as that three-part products is also GIVEN. There’s no need to discuss why those two formulas are true. The real work of the task is analyzing that three-part product in order to determine where it reaches its maximum. HINT: It is easy to maximize a cosine function with respect to the angle if the products in front of it are all actually constants with respect to that angle. Nearly every multivariable Calculus textbook actually presents a quick and dirty proof of Part B, so your job is to understand it and write it up with a lot more detail. The WGU textbook (check your Calc III Course of Study) covers this in Section 14.5. You might need to do some reading of prior sections if you’ve gotten rusty.

See the diagram in the link here of the key page, with my annotations.

This page states as factual what you need to carefully justify for the required proof in the task. I've drawn the question “WHY?” in the screenshot and pointed to every statement that needs to be fully explained to help you understand the difference between taking a statement on someone else's authority and actually explaining why it is true yourself.

**Task 3 Performance Task**

Complete-Submit the following task in TaskStream:

- Math Modeling & Connections: Task 3
For details about this performance assessment, see the "Assessment" tab in this course.
Mathematical Modeling and Connections Task 4: Classroom Geometry Perspectives

You will write about how a problem in geometry can be viewed from multiple perspectives. This ability is important, but it may not be present at first glance. Do you have a preference among the choices of vector, coordinate, and transformational perspectives when engaging in geometrical topics? Why do you suppose this is?

Classroom Geometry Perspectives

Consider the following questions as you complete this section:

- How are vector, coordinate, and transformational perspectives on the same geometric topic similar and different?
- Are you biased toward one perspective over another?
- What might you do to overcome that bias?
- Why might it be useful to be able to approach a problem from more than one perspective?

This topic addresses the following competencies:

**Competency 210.5.1: Mathematical Modeling**
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**Coordinate Systems and Analytical Geometry**

Review the relevant topics from the following course:

- College Geometry

Review Cartesian coordinates from the following course

- College Algebra

Revisit your prior notebook entries and reflect on how this knowledge can be applied now.

**Transformations in Geometry**

Review the relevant topics from the following course:

- College Geometry

Review matrix algebra from the following course:
• Linear Algebra

Revisit your prior notebook entries and reflect on how this knowledge can be applied now.

**Vector Geometry**

Review the relevant topics from the following course:

• College Geometry

Review vector basics from the following course:

• Linear Algebra

Revisit your prior notebook entries and reflect on how this knowledge can be applied now.

**Task 4 Preparation Technology: select one of the following to use for the task**

- DESMOS GRAPHING CALCULATOR (NO DOWNLOAD REQUIRED)
- GRAPHING IN MICROSOFT WORD WITH ADD-ON DOWNLOAD (PC ONLY)
- GRAPHER (MAC ONLY)
- GEOMETER'S SKETCHPAD WITH DOWNLOAD HERE
- GEOGEBRA WITH DOWNLOAD CLICK HERE

**Task 4 Guide**

Use the Assessment tab and the Preview button or log into TaskStream to read the task. It has you describe a problem that could be presented to students from three perspectives: coordinate systems, transformation, or vectors. Requires knowledge of College Geometry.

As if for a teacher colleague, explain how they should teach how to solve one specific geometry problem (Part A) from the perspectives of coordinate systems (analytical geometry, Part A1), transformations (transformational geometry, Part A2), and vectors (coordinate-free vector geometry, Part A3).

Don’t pick anything too complex for the geometry problem in Task 4 – some ‘obvious’ fact is fine, because the challenge is in doing it three separate ways. Usually a simple theorem from traditional synthetic Geometry is “do-able” in these three different ways. Make sure it is a nice basic proof such as perpendicularity of rhombus diagonals, congruence of rectangle diagonals, or congruence of isosceles triangle base angles, or some other such elementary Euclidean Geometry theorem.

No matter how basic the theorem you pick, you should expect one or two of the proofs to be easy-to-medium, while at least one is usually medium-to-hard. A key point is to carefully define the geometric objects before beginning any proofs. This is particularly important in transformational geometry, where most of the difficulty will likely be writing down the definition -- the results you want will often flow quite quickly from a clear and correct definition.

The best frame of mind to take when writing this task <and apologies in advance for using some
convenient stereotypes> is that your teacher colleague is a great guy, but he mostly loves his coaching job and he’s been teaching the lower-level math courses at your school for decades. He’s stepping in for a maternity leave next year and he has to cover a course he hasn’t taught and he’s forgotten most of his own advanced mathematics coursework. Your job this summer is a crash-course to prepare him to teach that course, and this task is one part of that training -- one that you have to provide him with all the written materials in advance because you’re going on vacation, but he can’t afford to fall behind.

A note on accidentally blending the three geometries: BE CAREFUL!

It is easy to use too much coordinate geometry within your vector and transformation proofs. For example, even using graph paper for vector and transformation perspectives is risky, because your other kinds of proofs should be done from a “pure” transformation or “pure” vector approach.

Vector geometry is supposed to refer only to lengths and angles and includes only the references of the positive x-axis and the unit length. Graph paper will likely push you to do a coordinate-based vector approach, and that is actually a disguised version of coordinate geometry and will therefore not be acceptable.

Transformation perspectives should also be explained with coordinate-free discussion, because rotations and reflections and the other fundamental transformations need only the two axes and the origin for references, and things ought to be done without a coordinate-based approach.

I have seen some people succeed when using coordinates for the other two approaches, but they made it clear it was a matter of convenience, not necessity -- a way to make the explanation more clear, not a way to skip or replace the pure transformation or pure vector explanations.

Task 4 Performance Task

Submit the following task in TaskStream:

- Math Modeling & Connections: Task 4

For details about this performance assessment, see the "Assessment" tab in this course.

Mathematical Modeling and Connections Task 5: Geometric Groups

In this task you will discuss how the symmetry group of a regular polygon is an example of the connection between abstract algebra and geometry. Does this seem intuitively obvious to you? Does it instead seem puzzling or mysterious?

Geometric Groups

Consider the following questions as you complete this section:
How can the geometric symmetries of a regular polygon be conceptualized as an abstract algebra group?
Why might you want to do this?
What implications does this connection have with regard to the interrelatedness of these two seemingly disparate mathematical branches?

This topic addresses the following competencies:

**Competency 210.5.1: Mathematical Modeling**
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**Regular Polygons**

Review the relevant topics from the following course:

- College Geometry

Revisit your prior notebook entries and reflect on how this knowledge can be applied now.

**Symmetry Groups**

Review the relevant topics from the following course:

- Abstract Algebra

Revisit your prior notebook entries and reflect on how this knowledge can be applied now.

**Task 5 Preparation Videos**

- COMPOSITION OF FUNCTIONS BASIC (8 min)
- INVERSE OF AN ELEMENT OF A GROUP (5 min)
- COMPOSITION OF FUNCTIONS ADVANCED (10 min)

**Task 5 Preparation Readings**

- EXAMPLE DIHEDRAL GROUP D6
- DEFINE VARIABLES D8
- PARTIAL TABLE FOR D8

**Task 5 Guide**

Use the Assessment tab and the Preview button or log into TaskStream to read the task. It has you write a general essay about relationships between Abstract Algebra and Geometry as well as connecting the symmetries of a square with a specific group. Requires knowledge of College Geometry and Abstract Algebra.
Part A. You have to write about connections between abstract algebra and geometry. You must include a discussion of symmetry as being part of both subjects; I recommend “The Beauty of Symmetry” by Annenberg Learner is one of many examples of explorations of that issue. The details of the rest of the discussion about connections between the courses is up to you, but a good starting point would be a firm understanding of what each course is about at the fundamental/conceptual level, and for this, a good reference is A Gentle Introduction to the Mathematics Subject Classification Scheme. Wikipedia and other encyclopedia-style references will add depth to that discussion for you.

Part A1. You have to discuss how a square’s symmetries are the same as a particular group — so you have to describe how all the possible ‘actions’ that create symmetry correspond to the elements of that group, and you have to describe how the interaction between those symmetric actions corresponds to a binary operation on the elements of that group.

One of the key issues is to demonstrate is the four necessary properties of being a group – most of the textbooks skip over that and just jump into the geometry without formally confirming the abstract algebra. You need to list the 4 rules of being a group, and then explicitly, separately, and carefully discuss why this set and this operation (the set of symmetries with the operation between them you’ve described) fulfill those four rules. the operation between elements is usually written *.

The point is, is that it is NOT a dihedral group until you PROVE it is a group. It’s just a “dihedral set with ways to interact elements of the set together.” You’re welcome to use any coherent combination of pictures, text, and Cayley tables, but it is best to address each of those 4 rules separately to increase the clarity of your writing.

Most students struggle with confirming associativity because their methods used for the first three rules becomes the requirement to do 1024 computations in order to confirm 512 equalities which ought to be organized into a “Cayley cube” -- NOT PRACTICAL! You need a different way to confirm that A*(B*C)=(A*B)*C. At this point, it is much more productive to investigate the connection between the operation * and the compositions of functions acting on squares. Think of it another way: A, B, and C are all functions that act on squares, and if you can explain that clearly, the problem boils down to showing function composition is associative, which takes a bit of algebraic manipulation and discussion of rules for applying functions, but is quite reasonable. You should be able to confirm that A?(B?C)(s)=(A?B)?C(s), where ? is the symbol for function composition and s represents an arbitrary element that those functions are acting on. Hint: functions act on elements right to left, so eventually both sides will boil down to the same thing.

Task 5 Performance Task

Submit the following task in TaskStream:

- Math Modeling & Connections: Task 5

For details about this performance assessment, see the "Assessment" tab in this course.
Axioms

In this task you will discuss how the axioms of a vector space can be rewritten using abstract algebraic structures as an example of the connection between linear algebra and abstract algebra. You may have guessed that these two branches were related because, after all, they are both forms of algebra. But it may surprise you what techniques from one area apply in the other.

Algebraic Axioms

Consider the following questions as you complete this section:

- How can the linear algebra axioms for a vector space be conceptualized by reformulating them using structures from abstract algebra?
- What advantages might this allow you to realize?
- What neat relationships lie hidden in these disciplines, waiting to be discovered?

This topic addresses the following competencies:

**Competency 210.5.1: Mathematical Modeling**
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**Competency 210.5.2: Mathematical Connections**
The graduate understands and can explain the wide applicability of linear and abstract algebra to other areas of mathematics, including the calculus and geometry.

Vector Spaces

Review the relevant topics from following course:

- Linear Algebra

Revisit your prior notebook entries and reflect on how this knowledge can be applied now.

Fields

Review the relevant topics from following course:

- Abstract Algebra

Revisit your prior notebook entries and reflect on how this knowledge can be applied now.

**Task 6 Preparation Videos**

- GROUPS - SHOWING G IS A GROUP - PART 1 (6 min)
- GROUPS - SHOWING G IS A GROUP - PART 2 (3 min)

**Task 6 Preparation Reading**

- EXAMPLE USING GROUP MODULUS-5 UNDER ADDITION
Task 6 Guide

Use the Assessment tab and the Preview button or log into TaskStream to read the task. It has you write about the connection between abstract algebra and linear algebra and use abstract algebra to shorten the list of axioms describing vector spaces. Requires knowledge of Linear Algebra and Abstract Algebra.

Part A. You have to write about the connection between abstract algebra and linear algebra.

FOUR WORD HINT: “DIFFERENT ADJECTIVES ON ALGEBRA”

You should certainly discuss the fact that a lot of Abstract Algebra examples use Linear Algebra. And you should certainly think about the fact that no two other courses from your major had the same root word with different adjectives -- you use algebra to solve problems in Calculus and in Geometry, but those courses aren’t called Geometric Algebra and Calculus Algebra -- so there’s got to be something special about Linear and Abstract that they share the same root word. A good starting point for this discussion and others would be a firm understanding of what each course is about at the fundamental/conceptual level, and for this, a good reference is A Gentle Introduction to the Mathematics Subject Classification Scheme. Wikipedia and other encyclopedia-style references will add depth to that discussion for you.

Part A1. Use abstract algebra to shorten the list of vector space axioms

FOUR WORD HINT: “ABELIAN GROUP NO RING”

The key idea is to use algebraic structural objects such as groups, rings, and fields to re-examine and shrink the long list of vector space axioms. You should start with the list of 10 axioms written in your Linear Algebra textbook and look for similarities to the rules for groups, rings, and fields written in your Abstract Algebra textbook.

This work is much simpler than it initially appears, because you can narrow down the list of possible algebraic structures quite quickly using the following point: the structures you studied in Abstract Algebra were all binary operators, meaning they took two elements from one set and created an element of that same set; that means anything that is not a binary operator from Linear Algebra cannot be described using Abstract Algebra. It might look similar, but it isn’t identical [Clarification: Advanced/Ambitious students might know that Module Theory is actually part of Abstract Algebra, so technically you could use it in this task and do a lot more work, but let’s consider that a topic for doctoral-preparation Abstract Algebra and therefore beyond the scope of what’s necessary for this task.]

Make sure to conclude your discussion by explicitly rewriting the new definition of a vector space that uses abstract algebra vocabulary and comment on how much shorter it is.

Task 6 Performance Task

Complete-Submit the following task in TaskStream:
• Math Modeling & Connections: Task 6

For details about this performance assessment, see the "Assessment" tab in this course.

**Final Steps**

Congratulations on completing the activities in this course! This course has prepared you to complete the assessment associated with this course. If you have not already been directed to complete the assessments, schedule and complete your assessments now.

**The WGU Library**

The WGU Library
The [WGU Library](https://www.wgu.edu/library) is available online to WGU students 24 hours a day.

For more information about using the WGU Library, view the following videos on [The WGU Channel](https://www.youtube.com/wgu):

- Introducing the WGU library
- Searching the WGU library

*Note: To download this video, right-click the following link and choose "Save as...": [download video](https://www.wgu.edu/library).*

**Center for Writing Excellence: The WGU Writing Center**

If you need help with any part of the writing or revision process, contact the Center for Writing Excellence (CWE). Whatever your needs—writing anxiety, grammar, general college writing concerns, or even ESL language-related writing issues—the CWE is available to help you. The CWE offers personalized individual sessions and weekly group webinars. For an appointment, please e-mail [writingcenter@wgu.edu](mailto:writingcenter@wgu.edu).

**Feedback**

WGU values your input! If you have comments, concerns, or suggestions for improvement of this course, please submit your feedback using the following form:

- [Course Feedback](https://www.wgu.edu/feedback)

**Accessibility Policy**
Western Governors University recognizes and fulfills its obligations under the Americans with Disabilities Act of 1990 (ADA), the Rehabilitation Act of 1973 and similar state laws. Western Governors University is committed to provide reasonable accommodation(s) to qualified disabled learners in University programs and activities as is required by applicable law(s). The Office of Student Accessibility Services serves as the principal point of contact for students seeking accommodations and can be contacted at ADASupport@wgu.edu. Further information on WGU’s Accessibility policy and process can be viewed in the student handbook at the following link:

- Policies and Procedures for Students with Disabilities