Course of Study Title: Mathematics Content (5-12), Part XI: Abstract Algebra

Description: This course of study presents the required sequence of learning steps and activities to help you develop competence in the subject area of Abstract Algebra. In this case, your competence will be assessed as you complete a series of performance tasks. The tasks are listed in the sequence below at the point in which you should have covered the learning necessary to build the necessary competence to successfully complete the task. Once all tasks are completed at the appropriate level of competence, you will receive a PASS on your AAP for the MRA Assessment. As with any learning activity, steps may be completed more quickly than noted below, or they could take the full amount of time indicated. We provide the pacing (Week One, Week Two, etc.) as a guide to the amount of time you should take to develop the competencies necessary and prepare to complete the required assessment on time. Completing your assessments within the required timeline keeps you on pace for Satisfactory Academic Progress and Graduation.

Introductory Text: The subdomain of MRA covers only five topics in seven tasks, but Abstract Algebra is a large subject area, with several other topics covered on the MBC objective exam. This component of your work at WGU is designed to help you to gain a broad overview of the field of Geometry with a fundamental understanding of some key concepts and principles.

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Office Hours.....................
Monday, 1-6 pm, Eastern Time Zone
Tuesday, 6-10 pm, Eastern Time Zone
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Thursday, 1-6 pm, Eastern Time Zone
Friday, 1-6 pm, Eastern Time Zone

Disposition Statement: Western Governors University supports the development and demonstration of professional teaching dispositions throughout the course of its Teachers College (TC) licensure programs. All TC students and faculty will demonstrate the following dispositions described in the Teachers College’s conceptual framework and code of ethics:

- Competent and caring
- Respectful and embracing of diversity
- Reflective practitioners
- Equitable and fair
- Professional practice consistent with the belief that all students can learn
- Collaborative professionals
- Professional leaders and change agents

Please review the <Teachers College Code of Ethics> found in the WGU Student Handbook.
Required Learning Resources (see listing on the resources tab of your AAP to enroll or order):

Order [Geneseo Algebra] and [EPGY Abstract] if you're new to Abstract Algebra or have forgotten it all. Order [EPGY Abstract] if you're reviewing or refreshing previously learned Abstract Algebra. Use [Goodman], an online textbook, as a supplement to the other resources, and use [Carter], software, if you need interactive geometric visualizations to learn the material.

- [EPGY Abstract]: *Modern Algebra – Self Study*, by Stanford EPGY, available free via your AAP Available LRs. It is CDs with Website access to additional materials.
- [Goodman] Goodman F. M. *Algebra: Abstract and Concrete* is online at [http://www.math.uiowa.edu/~goodman/algebrabook.dir/algebrabook.html](http://www.math.uiowa.edu/~goodman/algebrabook.dir/algebrabook.html). It takes a slow and detailed approach and will appeal to visual learners because many groups are explained geometrically and a few interactive geometric visualizations are available.
- [Carter] Group Explorer 2.0 is downloadable free software by Nathan Carter at [http://groupproject.sourceforge.net/](http://groupproject.sourceforge.net/). This software will let you interactively and visually explore the properties of groups.

Resources from Linear Algebra:


Appropriate Calculator: The TI-84+ graphing calculator, its predecessors TI-82, TI-83, or TI-83+, or equivalent calculators of other brand are recommended. Graphing calculators possessing built-in Computer Algebra Systems (CAS) are not allowed to be used on competency exams, so we recommend you do not use such a calculator while working on the mathematics tasks and topics. To download your TI screenshots to your computer, you will need a TI Connectivity Kit, available at [http://education.ti.com/educationportal/sites/US/productDetail/us_ti_connectivity_kit.html](http://education.ti.com/educationportal/sites/US/productDetail/us_ti_connectivity_kit.html)

PLEASE NOTE: The resources you are using to master the competencies for this assessment will also be valuable as you as you prepare for future assessments, namely, the MBC objective exam, the Praxis II exam, and any state-mandated mathematics content exams. Therefore, we recommend that you complete each activity contained in this document.

WEEK #1
Subject Title: Abstract Algebra
Subject Description: Introduction to abstract algebra
Background Information: These are not short topics! Spend plenty of time with the examples and exercises relating to these mathematical objects, and learn the ‘canonical examples’ for each.
Competency Title: Linear & Abstract Algebra
Numerical Code: 203.1.2
**Competency Description:** The graduate understands the basic concepts and important results in linear algebra and abstract algebra and understands the wide applicability of linear and abstract algebra to other areas of mathematics.

**Topic Title:** Groups, Rings & Fields  
**Instruction Text:** Tell whether a given mathematical object is a group, ring, field, or none of those by applying their definitions.

**Activity Title:** Rings & Fields  
**Activity Type:** Text Book Reading: [Geneseo Algebra]  
**Description:** Read Section 2.5 in [Geneseo Algebra]. Additionally, review supplemental reading earlier in Chapter 2.

**Activity Title:** Introductions to Groups  
**Activity Type:** Text Book Reading: [Geneseo Algebra]  
**Description:** Read Section 4.1 in [Geneseo Algebra]. Additionally, review supplemental reading later in Chapter 4.

**WEEK 2**  
**Subject Title:** Abstract Algebra  
**Subject Description:** Introduction to abstract algebra  
**Background Information:** These are not short topics! Spend plenty of time with the examples and exercises relating to these mathematical objects, and learn the ’canonical examples’ for each.  
**Competency Title:** Linear & Abstract Algebra  
**Numerical Code:** 203.1.2  
**Competency Description:** The graduate understands the basic concepts and important results in linear algebra and abstract algebra and understands the wide applicability of linear and abstract algebra to other areas of mathematics.

**Topic Title:** Critical Abstract Algebra Concepts, Ideas and Examples  
**Instruction Text:** Understand and apply important abstract algebra examples and ideas, such as identities, inverses, closure, isomorphisms, subgroups, subrings, ideals, subfields, cyclic groups, and integral domains

**Activity Title:** Online Reading & Exploration  
**Activity Type:** Content Review  
**URLs:** http://en.wikipedia.org/wiki/Abstract_algebra  
**Description:** Explore these topics in depth by starting with absorbing the ideas and main examples of Abstract Algebra shown in Wikipedia.

**Activity Title:** Wolfram’s Mathworld  
**Activity Type:** Exploration of Content  
**URLs:** http://mathworld.wolfram.com/AbstractAlgebra.html  
**Description:** Branch off into sub-fields and specialized examples by following links on those pages. Use the vocabulary in Topics 1 and 2 above to make sure you don’t go too far afield, but you will certainly need to explore a lot of material. Also use these links as sources for the specialized topics that appear in the tasks below [Abelian, automorphism, and modular arithmetic in particular].

**WEEK 3**
Subject Title: Abstract Algebra Proofs
Subject Description: Learn the requirements of and techniques for proving theorems in abstract algebra
Background Information:
Competency Title: Linear & Abstract Algebra
Numerical Code: 203.1.2
Competency Description: The graduate understands the basic concepts and important results in linear algebra and abstract algebra and understands the wide applicability of linear and abstract algebra to other areas of mathematics.

Topic Title: Theorem Proofs
Instruction Text: Prove given theorems of important results in abstract algebra

Activity Title: Proof-writing tips
Activity Type: Website Visit
URLs: http://mathforum.org/dr.math/faq/faq.proof.html
Description: Visit the Math Forum to find out important answers to the key questions about proofs: “What are Proofs? How do we understand Proofs? How do we write two-column proofs?”

Activity Title: Task 1 in MRA
Activity Type: Performance Task Assessment
Description: Prove a group with a given property is Abelian. Show how the equation defining a given property leads to the desired property of the group

WEEK 4
Subject Title: Automorphism
Subject Description: An automorphism is an isomorphism of a system of objects onto itself.
Background Information:
Competency Title: Linear & Abstract Algebra
Numerical Code: 203.1.2
Competency Description: The graduate understands the basic concepts and important results in linear algebra and abstract algebra and understands the wide applicability of linear and abstract algebra to other areas of mathematics.

Topic Title: Prove a function is an automorphism.
Instruction Text: Show that the function satisfies the properties necessary to be an automorphism.

Activity Title: Task 2 in MRA
Activity Type: Performance Task Assessment
Description: Be careful not to confuse the homomorphism's action as a function with the group operation's action as component-wise addition: (a, b) + (c, d) = (a+c, b+d) is component-wise addition where X=(a, b) and Y=(c, d) in R². How would you show the function is a morphism using X, Y, and X + Y?

WEEK 5
Subject Title: Modular Arithmetic
Subject Description: Modular arithmetic is an integer arithmetic system for integers. In Modular Arithmetic, numbers will "wrap around" back to the beginning after they reach as set high value. Clock arithmetic is one example of modular arithmetic.

Background Information:
Competency Title: Linear & Abstract Algebra
Numerical Code: 203.1.2
Competency Description: The graduate understands the basic concepts and important results in linear algebra and abstract algebra and understands the wide applicability of linear and abstract algebra to other areas of mathematics.

Topic Title: Mathematical Ring Proof
Instruction Text: Prove modular arithmetic is a ring.

Activity Title: Task 3 in MRA
Activity Type: Performance Task Assessment
Description: Show $\mathbb{Z}_n$ satisfies all the properties of a ring.

WEEK 6
Subject Title: Modular Arithmetic
Subject Description: Modular arithmetic is an integer arithmetic system for integers. In Modular Arithmetic, numbers will "wrap around" back to the beginning after they reach as set high value. Clock arithmetic is one example of modular arithmetic.

Background Information:
Competency Title: Linear & Abstract Algebra
Numerical Code: 203.1.2
Competency Description: The graduate understands the basic concepts and important results in linear algebra and abstract algebra and understands the wide applicability of linear and abstract algebra to other areas of mathematics.

Topic Title: Integral Domain Proof
Instruction Text: Prove modular arithmetic is an integral domain if and only if the modulus is prime.

Activity Title: Task 4 in MRA
Activity Type: Performance Task Assessment
Description: For an "if and only if" proof, the “if-then” must be done twice, once assuming the left side is true and showing the right side, and again, in the other direction.

WEEK 7
Subject Title: Abstract Algebra Relationships
Subject Description: Explain the relationship between abstract algebra and other fields of mathematics, such as geometry and linear algebra.

Background Information:
Competency Title: Linear & Abstract Algebra
Numerical Code: 203.1.2
Competency Description: The graduate understands the basic concepts and important results in linear algebra and abstract algebra and understands the wide applicability of linear and abstract algebra to other areas of mathematics.
**Topic Title:** Mathematical History

**Instruction Text:** History and mathematics of the non-constructible geometric shapes.

**Activity Title:** Task 5 in MRA 5

**Activity Type:** Performance Task Assessment

**Description:** MRA Task 5 (below) is an example of a connection between abstract algebra and geometry. These two problems are often referred to as "squaring the circle" and "trisecting the angle" and you'll find many references using Web searches. However, make sure to carefully evaluate the trustworthiness of your sources.

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**WEEK 8**

**Subject Title:** Abstract Algebra Relationships

**Subject Description:** Explain the relationship between abstract algebra and other fields of mathematics, such as geometry and linear algebra.

**Background Information:**

**Competency Title:** Linear & Abstract Algebra

**Numerical Code:** 203.1.2

**Competency Description:** The graduate understands the basic concepts and important results in linear algebra and abstract algebra and understands the wide applicability of linear and abstract algebra to other areas of mathematics.

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**Topic Title:** Vector Space Axioms

**Instruction Text:** Compression of vector space definition into four axioms.

**Activity Title:** Vector Spaces

**Activity Type:** Textbook Content Review


**Activity Title:** Vector Spaces

**Activity Type:** Textbook Content Review


**Activity Title:** Task 6 in MRA

**Activity Type:** Performance Task Assessment

**Description:** MRA Task 6 is an example of a connection between abstract algebra and linear algebra. “Special algebraic objects” mean groups, rings, and fields, sometimes with adjectives, for example, “division ring.”

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**WEEK 9**

**Subject Title:** Curriculum Relationships

**Subject Description:** Discuss the relationship between abstract algebra and the algebra in the grades 9-12 mathematics curriculum

**Background Information:**

**Competency Title:** Linear & Abstract Algebra
Numerical Code: 203.1.2
Competency Description: The graduate understands the basic concepts and important results in linear algebra and abstract algebra and understands the wide applicability of linear and abstract algebra to other areas of mathematics.

Topic Title: Algebra in the Curriculum
Instruction Text: Find the connections and relationships between the content that you have been learning about and pedagogical practices of secondary teachers.

Activity Title: “Seeing The Connections: Abstract Algebra & High School Mathematics”
Activity Type: Online Readings
URLs: http://www.smcm.edu/mathcs/Students/studentProjects/rmtalbot2005/index.html
Description: Rebecca Talbot’s Website “Seeing The Connections: Abstract Algebra & High School Mathematics”

Activity Title: High School Teacher Preparation
Activity Type: Online Readings
URLs: http://www.cbmsweb.org/MET_Document/chapter_5.htm
Description: Conference Board of the Mathematical Sciences, The Mathematical Education of Teachers Book:

Activity Title: Task 7 in MRA
Activity Type: Performance Task Assessment
Description: Secondary school algebra lesson plans about field properties. Make sure you focus on the conceptual development and communications skills of the students, not just computational expertise.

Feedback

If you wish to provide feedback on this course of study, please contact Rob Duncan, Mathematics Program Coordinator, at rduncan@wgu.edu.