Teachers College Course of Study  
Domain: Mathematics Content (5-12)  
Subdomain: Part V: Mathematical Modeling (MOA)

Course of Study Title: Mathematics Content (5-12) Part V: Mathematical Modeling

Description: This course outline presents the required sequence of learning steps and activities to help you develop competence in the subject area of Mathematical Modeling. In this case, your competence will be assessed as you complete a series of performance tasks. The tasks are listed in the sequence below at the point in which you should have covered the learning necessary to build the necessary competence to successfully complete the task. Once all tasks are completed at the appropriate level of competence you will receive a PASS on your AAP for the MOA Assessment. As with any learning activity, steps may be completed more quickly than noted below, or they could take the full amount of time indicated. We provide the pacing (Week One, Week Two, etc.) as a guide to the amount of time you should take to develop the competencies necessary and prepare to complete the required assessment on time. Completing your assessments within the required timeline keeps you on pace for Satisfactory Academic Progress and Graduation.

Introductory Text: This course outline presents the required sequence of learning activities to help you develop competence in the subject area of Mathematical Modeling. In this case, your competence will be assessed as you complete five performance tasks in TaskStream. This component of your work at WGU is designed to help you to gain a broad overview of the field of Mathematical Modeling with a fundamental understanding of some key concepts and principles.

Course Mentor ..................... Gideon Weinstein  
Email .............................. gweinstein@wgu.edu  
Telephone ......................... 1-866-895-9660, x1876  
Office Hours ...................... Monday, 9:00am – 4:00pm, Eastern Time  
........................................ Tuesday, 9:00am – 4:00pm, Eastern Time  
........................................ Wednesday, 9:00am – 4:00pm, Eastern Time  
........................................ Thursday, 9:00am – 4:00pm, 7:00pm – 12:00am, Eastern  
........................................ Friday, 9:00am – 4:00pm, Eastern Time

Disposition Statement: Western Governors University supports the development and demonstration of professional teaching dispositions throughout the course of its Teachers College (TC) licensure programs. All TC students and faculty will demonstrate the following dispositions described in the Teachers College's conceptual framework and code of ethics:

Competent and caring  
Respectful and embracing of diversity  
Reflective practitioners  
Equitable and fair  
Professional practice consistent with the belief that all students can learn  
Collaborative professionals  
Professional leaders and change agents
Please review the Teachers College Code of Ethics (http://kb.wgu.edu/display?n/articleDirect/index.asp?aid=1489&r=0.3862879) found in the WGU Student Handbook.

Learning Resources:
(See listing on the Available Learning Resources tab of your AAP to enroll or order)

Required:
PREREQUISITE KNOWLEDGE: You should have already passed MUC. Specifically, you should already know how to do scatterplots and lines of regression (Statistics) and how to compute integrals of rational functions by using partial fractions (Calculus). Note that MSA and MUA are not prerequisites because they cover topics that aren’t used in the MOA tasks. The Learning Resources you used to develop those competencies should be sufficient for doing the work described here, which is why they are all listed as optional.

Optional:
Blitzer, R. (2007). Algebra & Trigonometry, 3rd ed. Boston: Pearson Addison-Wesley. ISBN: 9780132191401. For materials at no cost to you, sign up for Blitzer using Available Learning Resources, which will give you free Web access to MyMathLab, which includes some videos, lots of quizzes, and other interactive capabilities as well as the full text of the book. This textbook is used mainly for other assessments in the Math 5-12 program, but has one section that is particularly useful for MOA.

Thomas, Jr., G. B., Weir, M. D., Hass, J, and Giordano, F. R. (2008). Thomas’ Calculus, Early Transcendentals, Media Upgrade, 11th ed. Boston: Pearson Addison-Wesley. ISBN: 9780321495754. For materials at no cost to you, sign up for Thomas’ Calculus using Available Learning Resources, which will give you free Web access to MyMathLab, which includes some videos, lots of quizzes, and other interactive capabilities as well as the full text of the book. This textbook is used mainly for other assessments in the Math 5-12 program, but has one section that is particularly useful for MOA.

Triola, M. F. (2006). Elementary Statistics, 10th ed. Boston: Pearson Addison Wesley. ISBN: 9780321331830. For materials at no cost to you, sign up for Triola using Available Learning Resources, which will give you free Web access to MyMathLab, which includes some videos, lots of quizzes, and other interactive capabilities as well as the full text of the book. This textbook is used mainly for other assessments in the Math 5-12 program, but has some sections that are particularly useful for MOA.

Appropriate Calculator: The TI-84+ graphing calculator, its predecessors TI-82, TI-83, or TI-83+, or equivalent calculators of other brands are recommended. Graphing calculators possessing built-in Computer Algebra Systems (CAS) are not allowed to be used on competency exams, so we recommend you do not use such a calculator while working on the mathematics tasks and topics. To download TI screenshots to your computer for inclusion in tasks, you will need a TI Connectivity Kit, available at http://education.ti.com/educationportal/sites/US/productDetail/us_ti_connectivity_kit.html.

PLEASE NOTE: The learning resources you are using to master the competencies for this assessment will also be valuable as you as you prepare for other assessments, namely, the Praxis II exam, and any state-mandated mathematics content exams. Therefore, we recommend that you fully utilize these resources.
WEEK 1
Subject Title: Getting Started
To successfully complete MOA, you should prepare a calendar to schedule times devoted to your studies. Share your calendar with family and friends so they are aware of your obligations.

Topic Title: Calendar Management
Use the brief outline below to help you organize when you will study what topics and how you will integrate the preassessments, formative tasks, exam, and summative tasks into your term. Important Note on Term Pacing: With effectively 24 weeks in a term, and this document covering roughly one-quarter of the CUs needed for a term, you should spend no more than 6 weeks on preparing for and passing the tasks and the exam, but those weeks need not be consecutive.

Some students work on two (or even three) quarter-term-sized areas at once, thus allowing for 12 (or 18) weeks of study of the two areas at once, therefore leaving more time for knowledge to “sink in,” while others prefer a single-minded focus on one area. If your term includes an objective exam, you might be able to use more time on these tasks and less preparing for the exam if that is necessary.

MOA Mathematical Modeling Calendar Outline
Week 1: Scatterplots, Correlation, and Lines of Regression
Week 2: Separable Differential Equations
Taskstream MOA Task 1, solving the initial model
SUGGESTION: Do Weeks 4 and 5 before Week 3 if you need more work on correlation
Week 3: Model Evaluation by simulating data loss
Taskstream MOA Task 2, re-running the initial model without the last data point
Week 4: Long-Term Behavior
Study Notebook Activity 1, computing behavior and correlation
Week 5: Model Evaluation by correlation and long-term behavior
Study Notebook Activity 2, discussing behavior and correlation
Week 6: Study Notebook Activity 3, developing an alternative model

Subject Title: Statistics Review
Subject Description: Two-variable data can be examined in different ways; using coordinate axis scatterplots and computing correlations and lines of regression is one way to do so.
Background Information: Statistical data analysis is a critical part of mathematical modeling.

Competency Title: Mathematical Modeling
Competency Description: The graduate knows how to use mathematics concepts to investigate patterns, make generalizations, formulate mathematical models, make predictions and validate results. (p100168)

Topic Title: Review Scatterplots, Correlation, and Lines of Regression
Instruction Text: Prepare to write a task to demonstrate your knowledge of mathematical modeling.

Activity Title: Read Sections 10.2 and 10.3 in Triola
Activity Type: Text
URLs: (www.coursecompass.com)
Description: The sections are titled “Correlation” and “Regression,” is particularly important. The examples on page 519 and the video animation on page 218 are important to understand, but you also need to learn how to get Excel to compute the regression line and correlation coefficient for you.

Activity Title: Practice Problems
Activity Type: Learning Activity
URLs: (www.coursecompass.com)
Description: Exercise 23 on page 537 and Exercise 25 on page 555 are particularly useful preparation for the task. Additionally, on CourseCompass they have interactive “Exercise” buttons, meaning you can access variations on those problems that include step-by-step solutions.

Activity Title: Optional: Using Microsoft Excel or the TI-84+
Activity Type: Text
URLs:
TI Activity Exchange: http://education.ti.com/educationportal/activityexchange/
Description: In the Microsoft sites, enter search terms “scatterplot” and “trendline” to get the relevant instructions. On the TI (Texas Instruments) site, to find the most relevant materials, click “Statistics” under the “Math” header, then click “Two-Variable Analysis.” A further refinement using the drop-down menus to limit the search to grades 9-12 and the TI-84-family calculator would also be useful. You’ll want to copy chart from Excel or the TI into your task to explain your work.

Activity Title: Optional: Read and interact with Websites by Wattenberg, Waner, and Stark
Activity Type: Text
URLs:
Wattenberg: http://www.math.montana.edu/frankw/ccp/modeling/discrete/snooping/learn.htm
Waner: http://people.hofstra.edu/faculty/Stefan_Waner/RealWorld/tutorialsf0/frame1_5.html
Stark’s SticiGui: http://www.stat.berkeley.edu/users/stark/SticiGui/Text/index.htm
Description: Wattenberg: A nice explanation of how to “snoop” population data to look for trends in growth rates.
Waner: A short and interactive tutorial on Linear Regression
Stark’s SticiGui: The text has a lot of interactive illustrations. Read Chapters 4 and 5, and pay particular attention to 5.2, 5.3, and 5.4.

WEEK 2
Subject Title: Differential Equations
Subject Description: When the derivatives of different functions are related to each other in an equation, that equation is called a differential equation. There are many techniques for finding the functions when only this differential equation about their derivatives is known.
Background Information: Solving differential equations is a critical part of mathematical modeling.
Competency Title: Mathematical Modeling
Competency Description: The graduate knows how to use mathematics concepts to investigate patterns, make generalizations, formulate mathematical models, make predictions and validate results. (p100168)

Topic Title: Separable Differential Equations
Instruction Text: Write a task to demonstrate your knowledge of mathematical modeling.

Activity Title: Read Section 9.1 in Thomas’ Calculus
Activity Type: Text
URLs: (www.coursecompass.com)
Description: The section is titled “Slope Fields and Separable Differential Equations.” Examples 3 and 4 are important to follow, and, on CourseCompass there is an interactive “You Try It” button to get you to practice solving similar examples, with the opportunity for line-by-line feedback.

Activity Title: Practice Problems
Activity Type: Learning Activity
URLs: (www.coursecompass.com)
Description: Exercises 9 through 18 on page 632 are particularly useful preparation for the task. Additionally, on CourseCompass there is an interactive “Exercise” button, meaning you can access variations on those problems that include step-by-step solutions.

Activity Title: Taskstream Task 1 in MOA (203.5.3-01)
Activity Type: Performance Task Assessment
Description: Summary: Do a scatterplot and linear trendline for absolute population growth in the USA rate vs. time for the past two centuries. Then use separation of variables to solve the differential equation described by the trendline to create a predictive model.

Activity Title: Note on MOA Tasks
Activity Type: Study Tip
Description: Unlike any other mathematics subdomain, the MOA tasks build upon one another, so after you find the model in this first task, the following tasks investigate its accuracy, long- and short-term
effectiveness, and explore alternate models. Because the answers to follow-up tasks depend on the answers to previous ones, you should allow for about a week between tasks so you can get confirmation you did the current one correctly before moving on. At the very least, make sure Task 1 is correct before doing those that follow.

Activity Title: Optional: Additional Readings
Activity Type: Online Text
URLs:
Dartmouth: http://www.math.dartmouth.edu/~klbooksite/3.03/303.html
Description:
SOS Math: Useful because the example is a rational function.
Wikipedia: Useful because one example requires partial fractions.
Dartmouth: Includes sample problems with links to fully-worked solutions.

WEEK 3 Subject Title: Evaluating and making predications with a mathematical model, part 1
Subject Description: Once a mathematical model has been developed, it can be used to make predictions about the real-world situation it models, but it also has to be evaluated for its usefulness at making accurate predictions.
Background Information: Evaluating a model is part of the mathematical modeling process, and the point of a model is to make predictions about the real world.
Competency Title: Mathematical Modeling
Competency Description: The graduate knows how to use mathematics concepts to investigate patterns, make generalizations, formulate mathematical models, make predictions and validate results. (p100168)

Topic Title: Model Evaluation by simulating data loss
Instruction Text: Write a task to demonstrate your knowledge of mathematical modeling.

Activity Title: Taskstream Task 2 in MOA (203.5.3-04)
Activity Type: Performance Task Assessment
Description: Summary: Compute short-term predictions and validate the model by rerunning it with less data.

WEEK 4 Subject Title: Evaluating and making predications with a mathematical model, part 2
Subject Description: Once a mathematical model has been developed, it can be used to make predictions about the real-world situation it models, but it also has to be evaluated for its usefulness at making accurate predictions.
Background Information: Evaluating a model is part of the mathematical modeling process, and the point of a model is to make predictions about the real world.
Competency Title: Mathematical Modeling
Competency Description: The graduate knows how to use mathematics concepts to investigate patterns, make generalizations, formulate mathematical models, make predictions and validate results. (p100168)
Topic Title: Long-term Behavior
Instruction Text: Write a task to demonstrate your knowledge of mathematical modeling.

Activity Title: Read Sections 2.4 and 2.5 in Thomas’ Calculus
Activity Type: Text
URLs: (wwwoursecompass.com)
Description: The sections are titled “One-Sided Limits and Limits at Infinity” and “Infinite Limits and Vertical Asymptotes.” Make sure you can follow the procedures when applied to exponential functions, such as Example 10 in 2.4 and Examples 10 and 11 in 2.5; these ideas will be particularly useful for looking at the long-term behavior of the population model you’ve developed.

Activity Title: Correlation Coefficient and Limit of P(t)
Activity Type: Study Notebook Activity 1
Note: This activity requires you to test your results from the Taskstream performance task for objective 203.5.3-01.

In completing part B of MOA Task 1 - 203.5.3-01, you were asked to do a linear regression of relative growth rate versus time.

A. Calculate the correlation coefficient r to four significant digits for your linear regression.

B. What is the long-term prediction of your model? Answer this question by calculating \( \lim_{t \to \infty} P(t) \) that you obtained in part C of MOA Task 1 - 203.5.3-01.
Justify your answer.

WEEK 5 Subject Title: Evaluating and making predications with a mathematical model, part 3
Subject Description: Once a mathematical model has been developed, it can be used to make predictions about the real-world situation it models, but it also has to be evaluated for its usefulness at making accurate predictions.
Background Information: Evaluating a model is part of the mathematical modeling process, and the point of a model is to make predictions about the real world.
Competency Title: Mathematical Modeling
Competency Description: The graduate knows how to use mathematics concepts to investigate patterns, make generalizations, formulate mathematical models, make predictions and validate results. (p100168)

Topic Title: Model Evaluation by correlation and long-term behavior
Instruction Text: Write a task to demonstrate your knowledge of mathematical modeling.

Activity Title: Online Reading
Activity Type: Text and reflection
URLs: http://en.wikipedia.org/wiki/Mathematical_modeling#Training
Description: In Wikipedia’s entry for "Mathematical model", read “Training” and “Model Evaluation” and then consider the following questions and activities:

1. What are the model parameters that you trained in the mathematical model for US population growth in MOA Task 1?
2. What do you think the relationship is between the correlation coefficient of a curve fitting and the accuracy of the resulting mathematical model?
3. How would you go about applying the "common approach to model evaluation" to the model in this Task?
4. Name several dates for which finding the population would be a matter of model interpolation.
5. Do model extrapolation in Task 1 to compute the predicted population in several different years. Do some short-term predictions for the early 21st century and some long-term predictions for the hundreds and thousands of years in the future. How accurate do you think these results are? What model parameter(s) is/(are) dominating the long-term results?

Activity Type: Study Notebook Activity 2

Description: Summary: Discuss the quality of the predictive model with respect to the correlation coefficient and the long-term behavior.

In the previous Study Notebook Activity, you calculated the correlation coefficient for your linear regression and the limiting value to argue for or against the mathematical model that you used in completing Taskstream Task 1 (203.5.3-01).

Write a brief essay (suggested length of 1–2 pages) in which you:
A. Explain the meaning of the correlation coefficient in interpreting the results of a linear regression (i.e., sign, magnitude).

B. Use the calculated correlation coefficient to argue for or against the mathematical model that you used in completing Taskstream Task 1 (203.5.3-01).

C. Use the limiting value to argue for or against the mathematical model that you used in completing Taskstream Task 1 (203.5.3-01).

WEEK 6
Subject Title: Developing alternative models
Subject Description: Once a mathematical model has been developed, it can be used to make predictions about the real-world situation it models, but it also has to be evaluated for its usefulness at making accurate predictions.
Background Information: Developing alternative models is an important part of the mathematical modeling process.
Competency Title: Mathematical Modeling
Competency Description: The graduate knows how to use mathematics concepts to investigate patterns, make generalizations, formulate mathematical models, make predictions and validate results. (p100168)
Topic Title: Developing an alternative model

Instruction Text: Write a task to demonstrate your knowledge of mathematical modeling.

Activity Title: Read 8.3 in Thomas’ Calculus
Activity Type: Text
URLs: (www.coursecompass.com)
Description: The section is titled “Integration of Rational Functions by Partial Fractions.” This technique will be necessary to solve the differential equation.

Activity Title: Optional: Read 8.3 in Blitzer
Activity Type: Text
URLs: (www.coursecompass.com)
Description: The section is titled “Partial Fractions.” This treatment is a bit slower and more detailed since it is a lower-level textbook, but it does not show the results when integrals are applied, which is a necessary part of solving the differential equation.

Activity Title: Alternative Model
Activity Type: Study Notebook Activity 3

Description: Summary: Do the modeling again with a change in basic assumptions. Your task in this objective is to formulate a new mathematical model for U.S. population growth.

To refresh your memory, please see Part I of Objective 203.5.3-01. It is not necessary to redo Part I for this objective.

A. Assume the relative growth rate is a linear function of population at time \( t \), i.e., \( r = b + aP \).
1. Do a scatter plot of relative growth rate versus population.
2. Fit a linear regression to the plotted points.
3. Create a graph of the following:
   a. The regression superimposed on the scatter plot (include a viewing window if using a graphing calculator)
   b. The equation for the regression with \( a \) and \( b \) accurate to at least four significant digits

B. In part A, you found \( a \) and \( b \) such that \( r = b + aP \). Use \( P_0 = 3.9 \) as your initial condition to find the particular solution for this differential equation. Note that you may find it easier to solve in terms of the constants \( a \) and \( b \). Show all steps in your solution.

Activity Title: Optional: Additional Readings
Activity Type: Online Text
URLs:
GraphPad Software, Inc: "Introduction"
http://www.curvefit.com/introduction.htm

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Comment [JG4]: Some elements of this equation did not copy over when this COS was converted to a Word document.
GraphPad Software, Inc: “You must pick the model”
http://www.curvefit.com/you_must_pick_model.htm
NIST/SEMATECH:
Description:
GraphPad Software, Inc: “A complete guide to nonlinear regression”
“Introduction” provides background.
“Choices” read only steps 1 through 3 to understand the idea.
“You must pick the model” to understand why alternative models
could be better.
NIST/SEMATECH: The “Engineering Statistics Handbook” contains a
slightly more mathematically-intensive approach to explaining how to
incorporate scientific knowledge into regression function selection.

Feedback
If you wish to provide feedback on this course of study, please contact Rob Duncan,
Mathematics Program Coordinator, at rduncan@wgu.edu.