Domain 202 — Mathematics Content (Grades 5-9)

MMA Mathematics Content (5-9) Part I: The MMA4 is a series of 20 mathematics tasks covering the following mathematics areas: Number Systems, Algebraic Structures, Geometry, Statistics, Probability, Precalculus, and Calculus. The tasks must be completed in TaskStream. Request that your mentor refer this assessment for you.

MMC Mathematics Content (5-9) Part II: The MMC4 is a proctored, computer-based assessment containing 43 items covering the same areas as the MMA. Time Limit: 2 hours. There is a pre-assessment available for this exam. Ask your mentor to refer you for the pre-assessment as you near completion of the relevant Learning Resources.

Past experience has indicated that relying on few resources for the Mathematics competencies may not be adequate preparation. While the Schaum's Outline series is comprehensive, please consider other study resources. The following collection of sites, online text books and other resources will be helpful in mastering the content objectives. The Mathematics Mentors have created this expanded study guide to better serve your needs and to give you resources to meet the needs of multiple learning styles. The materials listed below are not required, but consider reviewing them if you feel you need more preparation in those areas.

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PART V: Competency 202.4.2: The Calculus (P100160)
PART VI: MMA Tasks and Clarification

For each Competency, the Study Guide will 1) comment on the Learning Resources mentioned in the AAP, 2) note optional textbooks, including those described in the tasks, 3) list online textbooks, 4) list useful websites, and 5) provide links relevant to each of the objectives.

For the Tasks, the Study Guide will provide clarification, when appropriate.
Part I: Subdomain 202.1 — Number Systems & Algebraic Structures (s1060)

Competency 202.1.1: Number Systems & Algebraic Structures (P100156)

The graduate understands the real number system and its relationship to the complex number system; understands important algebraic structures; and understands the significance of functions in the study of number systems.

LEARNING RESOURCES:

Thinkwell's Calculus provides ILR with CD Roms and interactive website

Recommended Links:
http://mathworld.wolfram.com/ModularArithmetic.html
http://www.sosmath.com/algebra/algebra.html
http://www.purplemath.com/modules/index.htm
http://library.thinkquest.org/10030/algecon.htm?tqskip1=1

Online texts
http://www.themathpage.com/alg/algebra.htm
http://tutorial.math.lamar.edu/AllBrowsers/1314/1314.asp

Objectives

| 1. Define real number, complex number and imaginary number, and demonstrate accuracy in calculations with the elements of these number systems. | http://en.wikipedia.org/wiki/Complex_number
http://www.ping.be/~ping1339/complget.htm |
| --- | --- |
| 2. Explain how the complex number system is an extension of the real number system. | http://en.wikipedia.org/wiki/Complex_number
http://www.ping.be/~ping1339/complget.htm
http://www.ping.be/~ping1339/complget.htm |
| 4. Explain modular arithmetic and tell how to add and multiply in a given mod n. | http://en.wikipedia.org/wiki/Modular_arithmetic
http://www.math.rutgers.edu/~erowland/modulararithmetic.html |
<p>| 5. State the associative and commutative properties for | <a href="http://www.aamath.com/pro74b-propertiesmult.html">http://www.aamath.com/pro74b-propertiesmult.html</a> |</p>
<table>
<thead>
<tr>
<th><strong>Task</strong></th>
<th><strong>URLs</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>15. Define factor and multiple and provide examples of mathematical topics in grades 5-9 in which these concepts are central.</td>
<td><a href="http://www.shodor.org/interactivate/activities/">http://www.shodor.org/interactivate/activities/</a></td>
</tr>
<tr>
<td>16. Define prime and composite and explain how to prove a given theorem involving these concepts</td>
<td><a href="http://www.aaamath.com/g57-prime-or-composite.html">http://www.aaamath.com/g57-prime-or-composite.html</a> <a href="http://www.syvum.com/cgi/online/serve.cgi/squizzes">http://www.syvum.com/cgi/online/serve.cgi/squizzes</a></td>
</tr>
<tr>
<td>(such as “The only two consecutive primes numbers are 2 and 3”).</td>
<td>/math/prime.tdf</td>
</tr>
<tr>
<td>---------------------------------------------------------------</td>
<td>----------------</td>
</tr>
<tr>
<td>17. Define the term function and use the definition to show that a given relation either is or is not a function.</td>
<td><a href="http://en.wikipedia.org/wiki/Function_(mathematics)">http://en.wikipedia.org/wiki/Function_(mathematics)</a></td>
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</table>
[http://www(aaamath.com/B/est.htm](http://www.aaamath.com/B/est.htm)|
Part II: Subdomain 202.2 — Geometry (MS) (s1061)

Competency 202.2.1: Geometry (MS) (P100157)

The graduate understands synthetic, analytic, and transformational geometries and their relationship to measurement, and understands how geometry and measurement develop from intuitive investigations to formal arguments.

LEARNING RESOURCES:

Online Text Book
http://library.thinkquest.org/20991/geo/index.html

Objectives

http://en.wikipedia.org/wiki/Axiom
http://www.bymath.com/studyguide/geo/sec/geo5.htm
http://www.phil.cmu.edu/dschlimm/re.html
|---|---|
| 2. State the Pythagorean Theorem and its converse and use the theorem to solve a given problem. | http://www.cut-the-knot.org/triangle/pythpar/Fifth.shtml
http://www.cut-the-knot.org/triangle/pythpar/Drama.shtml
http://mathworld.wolfram.com/EuclidsPostulates.html
http://en.wikipedia.org/wiki/Parallel_postulate |
| 3. Derive a given formula from analytic geometry (such as the midpoint or distance formula). | http://www.absoluteastronomy.com/encyclopedia/P/Py/Pythagorean_theorem.htm
http://encyclopedia.laborlawtalk.com/Pythagorean_theorem#The_theorem
http://mathworld.wolfram.com/PythagoreanTheorem.html
http://www.usna.edu/MathDept/mdm/pyth.html
http://mathforum.org/isaac/problems/pythagthm.html |
| 4. Define each of the following transformations in the plane and show graphically their effect on a given planar figure (such as a triangle with given coordinates): transformations in the plane and show graphically their effect on given planar figure (such as a triangle with given coordinates): reflection, rotation, translation and dilation. (Note: appropriate information will be given here, such as "reflect about a particular line.") | www.k12_hi.us/~mathapl/im10sym.htm
www.ma.utexas.edu/users/gilbert/M333L/chp4vers4.pdf
www.e-zgeometry.com/study/glencoegeometry.htm
http://education.yahoo.com/homework_help/math_help/problem_list?id=minigeogt_8_1
<p>| 5. Prove a given theorem in Euclidean | <a href="http://www.jimloy.com/geometry/iso.htm">www.jimloy.com/geometry/iso.htm</a> |</p>
<table>
<thead>
<tr>
<th>(synthetic) geometry (such as “The base angles of an isosceles triangle are congruent”).</th>
<th><a href="http://www.monroe.k12.ms.us/hamilton/Robinson/notes.htm">www.monroe.k12.ms.us/hamilton/Robinson/notes.htm</a> <a href="http://www.math.washington.edu/~king/coursedir/m444a05/notes/02-PropertiesIsosTriangle.html">www.math.washington.edu/~king/coursedir/m444a05/notes/02-PropertiesIsosTriangle.html</a></th>
</tr>
</thead>
<tbody>
<tr>
<td>6. Perform a given geometric construction using a straight edge and compass and show that the construction yields the required result. (For example, “Bisect a given angle and show that the result yields the required bisection.”)</td>
<td><a href="http://www.coe.tamu.edu/~strader/geometry/bisectangle1.0/bisectangle.html">www.coe.tamu.edu/~strader/geometry/bisectangle1.0/bisectangle.html</a> <a href="http://constructions.homestead.com/index.html">http://constructions.homestead.com/index.html</a> <a href="http://www.jimloy.com/geometry/iso0.htm">http://www.jimloy.com/geometry/iso0.htm</a> <a href="http://www.estanciaonline.com/mathematics/geometry/Construction/angles/bisector.htm">www.estanciaonline.com/mathematics/geometry/Construction/angles/bisector.htm</a></td>
</tr>
<tr>
<td>9. Determine the equations of a line or plane given specific information pertaining to it (such as slopes, intercepts, etc.)</td>
<td><a href="http://www.wtamu.edu/academic/anns/mps/math/mathlab/int_algebra/int_alg_tut15_slope.htm">www.wtamu.edu/academic/anns/mps/math/mathlab/int_algebra/int_alg_tut15_slope.htm</a> <a href="http://www.okc.cc.ok.us/maustin/EquationsofLine/Equations%20of%20a%20Line.htm">www.okc.cc.ok.us/maustin/EquationsofLine/Equations%20of%20a%20Line.htm</a> <a href="http://www.algebra.com/algebra/homework/coordinate/lessons/Linear-Equations-II.lesson">www.algebra.com/algebra/homework/coordinate/lessons/Linear-Equations-II.lesson</a></td>
</tr>
<tr>
<td>12. Demonstrate the use of measurement to collect data, recognize relationships, and develop generalizations, including formulas.</td>
<td><a href="http://www.math.msu.edu/~nathsync/sketchmad/profiles/www.state.me.us/education/ires/math.htm">www.math.msu.edu/~nathsync/sketchmad/profiles/www.state.me.us/education/ires/math.htm</a> <a href="http://dimacs.rutgers.edu/nj_math_coalition/framework/ch01-04/ch01-04_09-12.html">http://dimacs.rutgers.edu/nj_math_coalition/framework/ch01-04/ch01-04_09-12.html</a></td>
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Part III: Subdomain 202.3 — Statistics & Probability (MS) (s1062)
The graduate understands and uses probability and statistics; knows the relationship between them and sampling and inference; and understands predictions.

Competency 202.3.1: Statistics & Probability (MS) (P100158)

LEARNING RESOURCES:
Schaum's Outline of Introduction to Probability and Statistics.

Additional Resources:

Online Text books
http://espse.ed.psu.edu/statistics/Chapters/contents1.html
http://davidmlane.com/hyperstat/

Additional Websites
http://www.cmu.edu/oli/index.html

Objectives

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<th>Objective</th>
<th>Resources</th>
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</table>
www.shepherd.edu/mathweb/probstat_guide.pdf
www.answers.com/topic/permutations-and-combinations
www.andrews.edu/~calkins/math/webtexts/prod02.htm
www.sunny.issaquah.wednet.edu/Mathclub02/13Lesson.pdf |
www.comp.leeds.ac.uk/andyr/teaching/ai32/prob.pdf
http://www.mathsnet.net/asa2/modules/s1.html#3
| 3 Calculate means, medians, and modes as measures of central tendency in a given set of data and explain their differing roles. | www.manatee.k12.fl.us/sites/elementary/palmasola/mathlabtutstat1.htm
http://academic.kellogg.edu/secristb/Math%20130/Course%20Notes/chapter3.htm
www.phschool.com/iText/math/sample_chapter/Ch02/02-07/PH_Alg1_ch02-07_Obj1.html |
| 4 Calculate and interpret standard deviations for a given set of data and | www.graphpad.com/articles/intercept_Analyzing_one_group/descr_stats.htm
www.blackwellpublishing.com/essentialmedstats/004.pdf
www.statcan.ca/english/edu/power/ch12/variance.htm
http://wind.cc.whecn.edu/~pwildman/statnew/section_5_.htm |
| **5. Organize a given set of data into a suitable form for interpretation of the data.** | [http://skyways.lib.ks.us/pathway/cameo/chap8.htm](http://skyways.lib.ks.us/pathway/cameo/chap8.htm)  
[www.nsdc.org/library/publications/jsd/letendre211.cfm](http://www.nsdc.org/library/publications/jsd/letendre211.cfm) |
|---|---|
[http://mathcentral.uregina.ca/QQ/database/QQ.09.04/brad1.html](http://mathcentral.uregina.ca/QQ/database/QQ.09.04/brad1.html)  
| **7. Discuss the role of simulation as a problem solving tool.** | [http://virtual-architecture.wm.edu/Telecollaboration/problemsolving.html#Structure-Simulations](http://virtual-architecture.wm.edu/Telecollaboration/problemsolving.html#Structure-Simulations) |
| **8. State the law of large numbers.** | [www.stat.berkeley.edu/~stark/SticiGui/Text/gloss.htm](http://www.stat.berkeley.edu/~stark/SticiGui/Text/gloss.htm)  
| **9. Describe the characteristics of a well-designed and well-conducted survey.** | [http://cls.coe.utk.edu/efftlc/support_developing_surveys.htm](http://cls.coe.utk.edu/efftlc/support_developing_surveys.htm)  
[http://www.managementhelp.org/evaluatn/questnrs.htm](http://www.managementhelp.org/evaluatn/questnrs.htm)  
[www.slais.ubc.ca/resources/research_methods/question.htm](http://www.slais.ubc.ca/resources/research_methods/question.htm) |
| **10. Use descriptive statistics to analyze a given set of data, make predictions, and make decisions.** | [www.amstat.org/publications/jse/secure/v8n3/mclean.cfm](http://www.amstat.org/publications/jse/secure/v8n3/mclean.cfm)  
[http://home.ubalt.edu/ntsbarsh/Business-stat/opre504.htm](http://home.ubalt.edu/ntsbarsh/Business-stat/opre504.htm)  
[www.amstat.org/publications/jse/v8n3_abstracts.html](http://www.amstat.org/publications/jse/v8n3_abstracts.html) |
| **12. Model and solve a problem that uses the mathematical expectation of a random variable (such as using fair coins, expected winnings, etc.).** | [http://www.dartmouth.edu/~chance/teaching_aids/books_articles/probability_book/amsbook.mac.pdf](http://www.dartmouth.edu/~chance/teaching_aids/books_articles/probability_book/amsbook.mac.pdf)  
[www.acad.sunytccc.edu/instruct/sbrown/stat/probsumm.htm](http://www.acad.sunytccc.edu/instruct/sbrown/stat/probsumm.htm)  
[www.stat.ucla.edu/~cochran/stat10/winter/lectures/lect8.html](http://www.stat.ucla.edu/~cochran/stat10/winter/lectures/lect8.html) |
**Subdomain 202.4 — Precalculus & the Calculus (s1063)**

The graduate understands and applies the principles of trigonometry; mathematical modeling; and logarithmic, exponential, polynomial and rational functions.

**Part IV: Competency 202.4.1: Precalculus (P100159)**

**LEARNING RESOURCES:**
Schaum's Outline of Introduction to Probability and Statistics.

**Online Text books**
http://www.themathpage.com/aTrig/trigonometry.htm
http://www.geocities.com/CapeCanaveral/Launchpad/2426/

**Additional Websites**
http://www.cmu.edu/oli/index.html

**Objectives**

<table>
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<tr>
<th>Objective</th>
<th>Resources</th>
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<tbody>
<tr>
<td>Topic</td>
<td>URL 1</td>
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<td>---------------------------------------------------------------------</td>
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<tr>
<td>4. Know the trigonometric function values of special angles (such as 0, 90 degrees, 60 degrees, etc.) and use these values to calculate the trigonometric functions of angles greater than 360 degrees.</td>
<td><a href="http://www.themathpage.com/aTrig/trigonometry.htm">http://www.themathpage.com/aTrig/trigonometry.htm</a></td>
</tr>
<tr>
<td>11. Solve given logarithmic and exponential equations.</td>
<td>See 9 and 10 sites</td>
</tr>
<tr>
<td>12. Explain how mathematical modeling is a problem solving tool by providing an example of a problem that uses modeling techniques for its solution.</td>
<td></td>
</tr>
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</tr>
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</table>
| http://argyll.epsb.ca/jreed/math9/strand2/2102.htm  
http://www.mtholyoke.edu/courses/jmorrow/math_models.html  
http://www.k-12prep.math.ttu.edu/journal/pedagogy/ostler01/article.pdf  
www.causascientia.org/math_stat/Tutorial.pdf  
http://www.tenet.edu/teks/mmacd/curric/wl/wltg.htm#_Toc434288124 |

<table>
<thead>
<tr>
<th>13. Solve a given quadratic inequality.</th>
</tr>
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</table>
| http://www.purplemath.com/modules/ineqquad.htm  
www.wtamu.edu/academic/anns/mps/math/matalb/col_algebra/col_alg_tut23_quadineq.htm  
http://www.gpc.edu/~jgutliph/Books/ia/quadratic_eqs_ineqs/quadratic_inequalities.htm  
http://www.math.unc.edu/Faculty/mccombs/web/alg/classnotes/inequalities/quadraticineq.html |
Part V: Competency 202.4.2: The Calculus (P100160)

The graduate has a conceptual knowledge of limit, continuity, differentiability, and integration; and applies these concepts to examples in mathematics and the sciences.

LEARNING RESOURCES:

Online Textbooks
http://www.webalt.net/Calculus-2004/
http://www.math.byu.edu/Math/CalculusBible/Text/pdfbook.pdf

Additional Websites
http://www.cmu.edu/oli/index.html

Objectives

| | http://www.math.gatech.edu/~cain/notes/calculus.html
| | www.algebralab.org/study aids/studyaid.aspx?file=Calculus_6-23.xml
| | http://www.math.scar.utoronto.ca/calculus/Redbook/
| | http://www.math.umn.edu/~garrett/calculus/first_year/notes.pdf
| | http://www.calculus.org/
| | http://www.webalt.net/Calculus-2004/

| 2. Find limits for given polynomial and rational functions (such as the limit as x approaches 0 for \((x - 2) / (x + 3)\). | http://www.math.umn.edu/~garrett/calculus/first_year/notes.pdf
| | http://www.calculus.org/
| | http://www.webalt.net/Calculus-2004/
| | http://www.math.byu.edu/Math/CalculusBible/Text/pdfbook.pdf
| | http://www.math.gatech.edu/~cain/notes/calculus.html
| | www.algebralab.org/study aids/studyaid.aspx?file=Calculus_6-23.xml
| | http://www.math.scar.utoronto.ca/calculus/Redbook/

| 3. Determine points of discontinuity for given functions on given domains (such as for the function \(f(x) = \tan(x)\) for a range containing \(\pi/2\) radians.) | http://www.math.byu.edu/Math/CalculusBible/Text/pdfbook.pdf
| | http://www.math.gatech.edu/~cain/notes/calculus.html
| | www.algebralab.org/study aids/studyaid.aspx?file=Calculus_6-23.xml
| | http://www.math.scar.utoronto.ca/calculus/Redbook/
| | http://www.math.umn.edu/~garrett/calculus/first_year/notes.pdf
| | http://www.calculus.org/
| | http://www.webalt.net/Calculus-2004/

<table>
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<tr>
<th>and integration rules to find derivatives of given algebraic functions (such as the derivative of ( f(x) = x^3 + 4x - 17 )) and the antiderivatives of given algebraic functions (such as the antiderivative of ( x^4 - 16x )).</th>
</tr>
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<tbody>
<tr>
<td>5. Approximate a given derivative numerically.</td>
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<tr>
<td>6. Approximate a given antiderivative numerically.</td>
</tr>
<tr>
<td>7. Explain the concepts of derivative and definite integral.</td>
</tr>
<tr>
<td>8. Solve given problems involving rates of change and optimization (including both mathematics and science problems.) (Note: problems here might involve finding the extrema for a given function; finding the maximum profit for a set of business data; finding the largest possible container given constraints; finding velocity and acceleration in a given situation, etc.)</td>
</tr>
<tr>
<td>9. Explain the relationship between the derivative and the slope of a curve; between the definite integral and the area under a curve.</td>
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</tbody>
</table>
Part VI: MMA Tasks and Clarification

Some notes about TaskStream Scoring:
1. Each task is scored 1, 2, 3, or 4 points in each of five to eight grading categories.
2. Scoring a 3 means you met the standard – you did what was expected, perfectly following directions. Scoring a 4 means you did exemplary work – you went above and beyond what was asked.
3. To pass, you must get a 3 or above in each grading category with the exception of at most a single score of 2; two or more 2s or any 1s means the task will be returned for revision.

<table>
<thead>
<tr>
<th>Common Scoring Categories for All Tasks</th>
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<tbody>
<tr>
<td>Communication</td>
</tr>
<tr>
<td>Mechanics (Grammar, punctuation, spelling, etc.)</td>
</tr>
<tr>
<td>Style (Word choice, sentence structure, etc.)</td>
</tr>
<tr>
<td>Overall Holistic.</td>
</tr>
</tbody>
</table>

Clarification
Your written work for all tasks must be easy to follow and understand, may have no more than 2 minor mechanical errors, and the sentence structure must be generally effective, the vocabulary solid, and the word choice accurate. Exemplary work must be extremely clear and easy to understand, have no mechanical errors, and have highly varied sentence structure, vivid vocabulary, and accurate word choice. 

The Overall Holistic score lets you know if the overall task is a pass [score of 3 or 4] or needs to be revised [score of 1 or 2].

General Process:
Look up terms you don’t recognize. Use the Web or perhaps an index from one of the recommended books.
Make an outline. Address all the issues brought up in the task description
Write the essay. Remember your goal is to answer the question completely and clearly. Be careful not to be too wordy. Stick tightly to the outline.
Edit the essay
Read your essay out loud to yourself. Yes, you will feel funny doing this. But it will help you spot errors you may have overlooked. Reread the question. Did you answer EVERY part? Check your calculations and/or illustrations twice.
Task One
Complex Numbers

Compare and contrast the real number system and the complex number system. Be sure to mention the parts of a complex number, and the operations of addition and multiplication. Discussions should include both algebraic and geometrical interpretations. Provide examples for each of the aforementioned situations. Your answer should be no more than two pages in length.

REFERENCES: References for this assignment are suggested materials only. You are NOT required to purchase them.


Clarification:
The question has many components. You must state how the two systems are the same and how they are different. The question has specifics you will need to include in this discussion (“parts of a complex number, operations of addition and multiplication, algebraic and geometrical interpretations”). Be certain you have covered each one of those topics and provided examples for each.

The scoring rubric states that an exemplary essay includes

• **BOTH** comparisons are made between how complex number multiplication is the same as using the distributive property with real number binomials AND contrasts are made with how the real plane is restricted to only scalar vector multiplication. *Two or more distinct examples are provided for each situation.*

• Comparisons are given as to how the operation of addition between real numbers and complex numbers **BOTH** algebraically AND graphically has the same one-to-one correspondence. *Two or more distinct examples are provided for each situation.*

• *Two or more distinct* examples of complex numbers are provided with at least one example showing how a complex number can be the same as a real number.

• Both real and imaginary parts are named and a definition of an imaginary number is given. *Mention is also given to when a complex number and real number are one in the same (i.e. \(2 + 0i = 2\))
Task Two
Modular Arithmetic

In an essay (3-5 paragraphs), explain modular arithmetic and how to add and multiply in mod 5. Be sure to include positive and negative integers and specific examples.

REFERENCES: References for this assignment are suggested materials only. You are NOT required to purchase them.


Clarification:
The question requests that you explain what modular arithmetic is and then provide 6 examples in mod 5. Make sure you have three examples that show multiplication (with negative numbers). Also, check that you have three examples that show addition (with negative numbers).

The scoring rubric states that an exemplary essay includes
• No calculation errors
• At least 3 examples of addition including negative numbers
• At least 3 examples of Multiplication including negative numbers
Task Three
Inverse Functions

Write an essay (250-500 words) discussing inverses. Be sure to discuss one-to-one, the step-by-step process of finding an inverse, examples of functions with and without inverses, and the graphic results of a function and its inverse. Pictures and/or graphs may be helpful in your essay.

REFERENCES: References for this assignment are suggested materials only. You are NOT required to purchase them.


Clarification:
The question asks that you explain the horizontal line test (this may be a good place for a graph). Show the step-by-step process of finding an inverse. Then be sure to graph the function, it’s inverse and the y=x line. Discuss why the functions are symmetrical about the x=y axis. Give examples of functions that do and do not have inverses. Explain why they do or do not have inverses.

The scoring rubric states that an exemplary essay includes
- One to one: Discusses horizontal line test including that for every y value there is only one x value
- Process: Includes a) determining one-to-one, b) switch x and y, c) solve for y, d) verify that f(f⁻¹(x)) = x and f⁻¹(f(x)) = x
- Examples: Gives examples of both functions with and without inverses and discusses why they do or do not have an inverse.
- Graphs both functions correctly and y = x and discusses symmetry
In a brief essay (3-5 paragraphs), define odd and even numbers both symbolically and using everyday language.

REFERENCES: References for this assignment are suggested materials only. You are NOT required to purchase them.


**Clarification:**
The question has two components. The first component is the symbolic definition of even and odd numbers. Be careful to explain the symbolic definition clearly. The second component is the everyday-language definition. Be sure to show four examples and talk about why the example is even or odd.

The scoring rubric states that an exemplary essay includes
- **Symbolic definition:** Definition is correct and includes a clear explanation
- **Everyday definition:** Explanation is correct and clear and includes at least 4 specific examples
Task Five
Mathematical Topics

Define “factor” and “multiple” and provide 4 examples of mathematical topics in grades 5-8 in which these concepts are central.

REFERENCES: References for this assignment are suggested materials only. You are NOT required to purchase them.

Rosen, Kenneth, Elementary Number Theory p.31.

Clarification:
The question has two components. The first component is the definition of factors and multiples. The second component asks for four examples of topics that uses these concepts. Be careful, this is not asking for example problems, but topics. It may help you to ask yourself, “What could I not do if I didn’t know how to use factors and multiples?”

The scoring rubric states that an exemplary essay includes
• Definitions are correct, complete, and clear.
• 4 examples are given
Task Six
Prime and Composite Numbers

Define prime and composite and explain how to prove a theorem involving these concepts.

REFERENCES: References for this assignment are suggested materials only. You are NOT required to purchase them.

Rosen, Kenneth, Elementary Number Theory, p. 66.

Clarification:
The question has two components. The first component is the definition of prime and composite numbers. Make sure you included examples for each type of number. The second component asks for a proof that involves these concepts.

The scoring rubric states that an exemplary essay includes

• Prime-- Definition is accurate with examples
• Composite-- Definition is accurate with examples
• Proof is correct, concise, complete, with no errors
Task Seven
Truncation and Rounding

Explain truncation and rounding. Include several examples of their application in grades 5-8 mathematics.

Clarification:
The question has two components. The first component is the definition of truncation and rounding. The second component is the examples of application. Be careful, the task wants examples of application not examples of truncating and rounding.

The scoring rubric states that an exemplary essay includes
• Definitions are correct, complete, and clear.
• 4 examples are given
Task Eight
Distance Formula

Create a project deriving the formula for the distance between two points in analytic geometry. Describe all terms used as well as listing the steps. Include a diagram or graph. Discuss how the Pythagorean theorem helps to find distances in 2 and 3 dimensions. Apply the distance formula to real-life examples. List any references used in a bibliography.

REFERENCES: References for this assignment are suggested materials only. You are NOT required to purchase them.


Clarification:
The question has four components. The first component is the formula. Make sure you define all of the terms used including point, plane, and line segment. Show how the formula was derived. Have a clearly labeled diagram to accompany your explanation. The second component is the Pythagorean Theorem. Discuss how the theorem finds distances in 2 and 3D. Discuss how the formula in part one is related to the Pythagorean Theorem. The final component is real-life applications. Show how the formula is used in real life. Check books, journals on ERIC and WebPages. The fourth component is the bibliography. Make sure all of your references are cited using APA format.

The scoring rubric states that an exemplary essay includes

- Formula--Defines terms used, point, plane, line segment, proof has easy to follow steps, references axioms used, diagram labeled Formula well explained, task stated clearly
- Explains how the Pythagorean Theorem helps to find distance in 2D and 3D and Relates Pythagorean theorem to derived formula, diagrams clear
- Several Real-life examples given and distance formula applied with clear diagrams
- Bibliography contains Sources from a mixture of books, journals, web pages, and dissertations
GIVEN the following theorem: The base angles of an isosceles triangle are congruent.

Create a lesson plan to prove the above theorem in Euclidean (synthetic) Geometry. The lesson plan should include the lesson title, purpose of the lesson, the lesson objectives, pre-instructional techniques, instructional procedures and strategies including the proof, questions the teacher will ask students, questions the teacher anticipates from students, and bibliography.

REFERENCES: References for this assignment are suggested materials only. You are NOT required to purchase them.


**Clarification:**
The question has eight components. First title your lesson. Then clearly state the purpose of the lesson and the lesson objectives. Next state the pre-instructional techniques. This includes the definition of terms used. Next provide step-by-step instructions for how you would conduct the lesson. Write the instructions so that they could be duplicated by another teacher. In this section include the proof. Make sure the proof is clearly explained and the steps are listed. Clearly label the diagrams. Also write the questions both you and the student would ask. Try to make these questions engaging and intended to stimulate conversation. Answer each of the student-questions clearly.

The scoring rubric states that an exemplary essay includes

- Objective defined, definitions and terms reviewed (triangle, isosceles triangle, line, congruent, angle), clear understanding of the task conveyed
- Proof steps listed, easy to follow and has good understanding, axioms and observations given Technique well explained, diagrams clear
- Diagram clear, parts of triangle labeled correctly Technique well explained
- Feed back Able to encourage students’ thinking Questions for and from the students to summarize and clarify task
**Task Ten**  
**Geometric Construction**

**TASK STATEMENT:** Create a chart describing how to bisect a given angle and show that the result yields the required bisection. List the steps necessary to draw and bisect the angle using a straight edge and compass, and define the terms used in this task. List any references used in a bibliography.

**REFERENCES:** References for this assignment are suggested materials only. You are NOT required to purchase them.


**Clarification:**
The question has three components. The first component is the chart describing how to bisect a given angle and the proof that your result is the required bisection. Use well-drawn diagrams to augment your description. The second component is the list of the steps needed to draw and bisect an angle. As you list these steps be certain to define each of the terms you use. Draw a diagram and label it clearly.

The scoring rubric states that an exemplary essay includes:

- Terms defined (angle, arc, vertex, intersect (sides), radius), clear understanding of task conveyed
- Steps listed, angle drawn, compass is placed on vertex, compass on intersections, arcs or circles drawn and Technique well explained, diagrams clear, radius measured
- Diagram clear, angle, vertex, arcs, drawn and labeled, steps marked and Technique well explained, diagrams clear
Task Eleven
Measurement

Create a project to demonstrate the use of measurement to apply approximate techniques, tools and formulas to determine measurement by measuring different attributes of a bucket including weight, volume/capacity, length and area. Describe the unit of measures used to collect the measurement data, as well as measurement instruments used. Identify relationships between the attributes being measured and the unit of measure used. Develop generalizations including formulas that can be used for determining measurements. Justify the role of estimation in the learning measurement. Include references in a bibliography.

<table>
<thead>
<tr>
<th>Scoring Category</th>
<th>Adequate</th>
<th>Excellent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bucket attributes</td>
<td>Discusses weight, volume, length and area units</td>
<td>Discusses measurement language, various units of measurement</td>
</tr>
<tr>
<td>Units/Instruments used</td>
<td>Discusses comparisons of measure as length to a ruler, etc., charts or diagrams</td>
<td>Explains more than one type of unit or way in which to measure each attribute</td>
</tr>
<tr>
<td>Relationships</td>
<td>Discusses numeric relationship between what is being measured and the unit as well as relationships between all measurements of the task</td>
<td>Clearly explains how measurements are related; includes diagrams or charts</td>
</tr>
<tr>
<td>Generalizations/formulas</td>
<td>Develops generalizations and formulas for the task attributes</td>
<td>Clearly stated and easy to follow</td>
</tr>
<tr>
<td>Estimation</td>
<td>Discusses estimation's focus on attribute being measured and the measuring process.</td>
<td>Defines estimating strategies and importance of estimating before measuring.</td>
</tr>
<tr>
<td>Bibliography</td>
<td>Adequate reference</td>
<td>Several references from different types of media sources used</td>
</tr>
</tbody>
</table>
Write an essay of 500 words explaining how measurements are approximations and how all measurements include some error. Discuss how an awareness of precision due to unit size and the need for precision in different situations is an important aspect of measurement. Explain how these differences in units affect precision. Give some examples when precision is not required and some examples when precision is significant. Include any references in a bibliography.

### Scoring Category

<table>
<thead>
<tr>
<th>Approximations</th>
<th>Adequate</th>
<th>Excellent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Clear explanation of what it means to measure and why it is approximate</td>
<td>Discusses measurement language, size of units, comparisons, etc.</td>
</tr>
<tr>
<td>Errors</td>
<td>Explains measurement is precise to one-half the smallest unit used in measurement</td>
<td>Defines errors as problems in measurement as well as instruments used</td>
</tr>
<tr>
<td>Precision</td>
<td>Discusses that the smaller the unit the greater degree of precision, and precision depending upon the situation</td>
<td>Explains how precision to size of unit is important in all measuring tasks.</td>
</tr>
<tr>
<td>Examples</td>
<td>Real-life examples explained as to their need for precision</td>
<td>Uses many ways to describe measurement.</td>
</tr>
<tr>
<td>Bibliography</td>
<td>Adequate reference</td>
<td>Several references from different types of media sources used</td>
</tr>
</tbody>
</table>
You are given the responsibility of presenting the results of a yearlong study of a teaching strategy to the school board. Fifty-two students were chosen at random from the school population to participate in the study. Twenty-six students were then randomly chosen to serve as a control group and received traditional instruction in the subject. The other 26 experienced the alternative treatment. The following table gives the post-test results for the 52 students.

<table>
<thead>
<tr>
<th>Control Group</th>
<th>Treatment Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>71</td>
<td>65</td>
</tr>
<tr>
<td>81</td>
<td>75</td>
</tr>
<tr>
<td>98</td>
<td>75</td>
</tr>
<tr>
<td>62</td>
<td>63</td>
</tr>
<tr>
<td>65</td>
<td>74</td>
</tr>
<tr>
<td>81</td>
<td>75</td>
</tr>
<tr>
<td>77</td>
<td>98</td>
</tr>
<tr>
<td>74</td>
<td>58</td>
</tr>
<tr>
<td>65</td>
<td>74</td>
</tr>
<tr>
<td>89</td>
<td>45</td>
</tr>
<tr>
<td>78</td>
<td>81</td>
</tr>
<tr>
<td>89</td>
<td>65</td>
</tr>
<tr>
<td>78</td>
<td>74</td>
</tr>
</tbody>
</table>

Prepare a report to submit to the school board.
• Use the most appropriate statistical test to present the information. (Hint: Consider using a "difference of means" test.)
• Present the information in both a graphic (table, graph, etc.) and a written form (one to two paragraphs).
• Summarize and make a recommendation based on the results (one to two paragraphs).

REFERENCES: References for this assignment are suggested materials only. You are NOT required to purchase them.
Prepare an essay of six to ten paragraphs on the role of simulation in statistical problem solving. Include examples of when simulation needs to be used, briefly describe the methodology of simulation, and discuss limitations of using simulations.

REFERENCES: References for this assignment are suggested materials only. You are NOT to required to purchase them.


<table>
<thead>
<tr>
<th>Scoring Category</th>
<th>Adequate</th>
<th>Excellent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Components</td>
<td>All three components included (examples, methodology, limitations).</td>
<td>Discussed all required components and included introductory and closing paragraphs.</td>
</tr>
<tr>
<td>Uses of simulation</td>
<td>Describes when simulation is necessary and provides one example.</td>
<td>Describes when simulation is necessary and provides more than one example.</td>
</tr>
<tr>
<td>Methodology of simulation</td>
<td>Includes two of the components listed in the key.</td>
<td>Includes all of the components listed in the key.</td>
</tr>
<tr>
<td>Limitations</td>
<td>Discusses the difference between the model and the simulation and explores the introduction of another layer of randomness (i.e., the sample).</td>
<td>Also gives ways to assess the uncertainty of the simulation.</td>
</tr>
</tbody>
</table>
A game company that manufactures a dart game that dispenses tickets has been asked to put a business plan together for a customer who wishes to buy the game. The dart game gives 5 tickets for a score of 50, 4 tickets for a score of 40, 3 tickets for a score of 30, two tickets for a score of 20, 1 ticket for a score of 10, and no tickets if the board is not hit.

The probabilities of getting each score are contained in this table:

<table>
<thead>
<tr>
<th>Score</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>(\frac{1}{32})</td>
</tr>
<tr>
<td>40</td>
<td>(\frac{1}{16})</td>
</tr>
<tr>
<td>30</td>
<td>(\frac{1}{8})</td>
</tr>
<tr>
<td>20</td>
<td>(\frac{1}{4})</td>
</tr>
<tr>
<td>10</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td>0</td>
<td>(\frac{1}{32})</td>
</tr>
</tbody>
</table>

The customer will be exchanging the tickets for stuffed animals that cost $3.00 each. Prepare a report for the customer that presents the expected value in points of each throw of the darts, and then present a graphic representation of at least three plans relating cost per dart, cost in tickets of the animals, and expected profit. Include a one or two paragraph summary of the possible plans.

REFERENCES:
References for this assignment are suggested materials only. You are NOT required to purchase them.

Task Sixteen
Compound Interest

An investment advisor is planning the retirement of a customer. The customer has $10,000 dollars to invest, and needs the investment to increase to $30,000 within 20 years. The report should include three sections. First, an introductory paragraph that reviews the customer’s requirements for the investment plan. Second, three different investment plans including at least one example of an actual investment, the average growth rate over the past several years, and the time required to increase the investment to $30,000. Finally, a concluding paragraph that summarizes the three investment plans and makes a recommendation.

REFERENCES:
References for this assignment are suggested materials only. You are NOT required to purchase them.


<table>
<thead>
<tr>
<th>Scoring Category</th>
<th>Adequate</th>
<th>Excellent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction and conclusion</td>
<td>Both elements included</td>
<td>Both elements included, writing clear, concise, and well prepared</td>
</tr>
<tr>
<td>Investment Plan</td>
<td>Three plans proposed, meets customer needs, actual stocks or funds quoted for all plans.</td>
<td>Three plans proposed, meets customer needs, more than one actual stock or fund quoted for all plans.</td>
</tr>
<tr>
<td>Accuracy of Calculations</td>
<td>All calculations accurate</td>
<td>All calculations accurate presented in a clear easy to read format</td>
</tr>
</tbody>
</table>
Task Seventeen
Trigonometric Identity

Prove that \[ \frac{\sin \theta}{1 - \cos \theta} - \frac{1 + \cos \theta}{\sin \theta} = 0 \]

Provide reasons (identities, operations, etc.) for each step in the proof. Write a short (one or two paragraph) summary of the solution. Include any thoughts, ideas or strategies used to prove the identity. Also discuss any difficulties encountered and how those difficulties were overcome.

REFERENCES:
References for this assignment are suggested materials only. You are NOT required to purchase them.


<table>
<thead>
<tr>
<th>Scoring Category</th>
<th>Adequate</th>
<th>Excellent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proof</td>
<td>All steps included, with no gaps between steps</td>
<td>All steps included, no extra steps left in final solution</td>
</tr>
<tr>
<td>Reasons</td>
<td>All reasons accurate</td>
<td>All reasons accurate including descriptions of all algebra steps</td>
</tr>
<tr>
<td>Summary</td>
<td>Summary includes three sample elements</td>
<td>Summary includes four sample elements</td>
</tr>
</tbody>
</table>
Research and create functions to explain the concepts of limit and continuity.

1. Create a function, f(x), and pick a point c such that the limit of f(x) as x approaches c from the right and the limit of f(x) as x approaches c from the left are equal and the function is continuous. Show the values of the limits and explain why the function is continuous. The explanation should be intuitive as well as mathematical. Include a graph of the function.

2. Create a function, f(x), and pick a point c such that the limit of f(x) as x approaches c from the right and the limit of f(x) as x approaches c from the left are equal, the function is defined at the point c but the function is not continuous at c. Show the values of the limits and explain why the function is not continuous. The explanation should be intuitive as well as mathematical. Include a graph of the function.

3. Create a function, f(x), and pick a point c such that the limit of f(x) as x approaches c from the right and the limit of f(x) as x approaches c from the left are equal, the function is not defined at the point c and the function is not continuous at c. Show the values of the limits and explain why the function is not continuous. The explanation should be intuitive as well as mathematical. Include a graph of the function.

4. Create a function, f(x), and pick a point c such that the limit of f(x) as x approaches c from the right and the limit of f(x) as x approaches c from the left are not equal in value. Explain why the limit of f(x) as x approaches c does not exist and why the function cannot be continuous. The explanation should be intuitive as well as mathematical. Include a graph of the function.

REFERENCES:
References for this assignment are suggested materials only. You are NOT required to purchase them.


<table>
<thead>
<tr>
<th>Scoring Category</th>
<th>Adequate</th>
<th>Excellent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical Accuracy</td>
<td>One mathematical error, but functions still satisfy the requirements.</td>
<td>No mathematical errors</td>
</tr>
<tr>
<td>Choice of Functions</td>
<td>Three functions satisfy the given requirements</td>
<td>All functions satisfy the given requirements.</td>
</tr>
<tr>
<td>Mathematical Explanations</td>
<td>Mathematical explanations are adequate.</td>
<td>All mathematical explanations extremely clear and easy to understand.</td>
</tr>
</tbody>
</table>
Calculus grew out of four major problems that European mathematicians were working on during the seventeenth century:

1. The tangent problem.
2. The velocity and acceleration problem.
3. The maximum and minimum problem.
4. The area problem.

Write an essay about how derivatives and the definite integral are used to solve the above problems. Specifically, the following items must be included:

1. How does the tangent line evolve from the secant line and how does the derivative relate to the tangent line?
2. How are derivatives used to solve velocity and acceleration problems?
3. How are derivatives used to solve maximum and minimum problems?
4. How is the definite integral used to solve area problems?

This essay should be no more than two typewritten pages using a legible 12 point font.

REFERENCES:
References for this assignment are suggested materials only. You are NOT required to purchase them.


<table>
<thead>
<tr>
<th>Scoring Category</th>
<th>Adequate</th>
<th>Excellent</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tangent Problem</strong></td>
<td>Adequate description of how the tangent line evolves from the secant line and how the derivative relates to the tangent line.</td>
<td>High level description of how the tangent line evolves from the secant line using limits and how the derivative relates to the tangent line.</td>
</tr>
<tr>
<td><strong>Velocity and acceleration problem</strong></td>
<td>Adequate explanation about the relationships between derivatives and position, velocity, and acceleration</td>
<td>High level discussion of the relationship.</td>
</tr>
<tr>
<td><strong>Maximum and Minimum problems</strong></td>
<td>Adequate explanation of how to use the first derivative to find maximum and minimum points.</td>
<td>High level discussion about using first derivative to find maximum and minimum points and use them to solve problems.</td>
</tr>
<tr>
<td><strong>Area problems</strong></td>
<td>Adequate explanation of the relationship between the definite integral and area under the curve.</td>
<td>High level discussion of the definite integral and the area under the curve and its use in problem solving.</td>
</tr>
</tbody>
</table>
Write an essay to explain the relationships between the derivative and the slope of a curve, and between the definite integral and area under the curve. Specifically, the following items must be included:

1. A discussion of the relationship between the slope of a secant line, the slope of a tangent line and the derivative.
2. A discussion of the relationship between the area of a finite number of rectangles under a curve and an infinite number of rectangles under a curve and the definite integral.

This essay should be no longer than one typewritten page written in a legible 10 or 12 point font.

REFERENCES:
References for this assignment are suggested materials only. You are NOT required to purchase them.


<table>
<thead>
<tr>
<th>Scoring Category</th>
<th>Adequate</th>
<th>Excellent</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Derivative</td>
<td>Adequate discussion of why the derivative is the instantaneous rate of change.</td>
<td>A clear and high level discussion of how the slope of the secant line, slope of the tangent line and the derivative are related.</td>
</tr>
<tr>
<td>The Definite Integral</td>
<td>Adequate discussion about why the definite integral gives the area under a curve.</td>
<td>A clear and high level discussion of how the definite integral is the limit of the sum of the areas of rectangles under the curve.</td>
</tr>
</tbody>
</table>