MDA Mathematics Content (5-12) Part I: The MDA4 is a series of mathematics tasks covering the following mathematics areas: Calculus, Analysis, Probability, Statistics, and Discrete Mathematics. The tasks must be completed in TaskStream. Request that your mentor refer this assessment for you.

MDC Mathematics Content (5-12) Part II: The MDC4 is a proctored, computer-based assessment containing 23 items covering the following mathematics areas: Calculus, Analysis, Probability, Statistics, and Discrete Mathematics. Time Limit: 1.5 hours. There is a pre-assessment available for this exam. Ask your mentor to refer you for the pre-assessment as you near completion of the relevant Learning Resources.

**TABLE OF CONTENTS**

PART I: Competency 203.3.1: Statistics & Probability (HS) (P100165)

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PART III: Competency 203.5.1: Symbolic Logic & Set Theory (P100167)

PART IV: Competency 203.5.2: Induction & Recursion (P100163)

PART V: Competency 203.5.3: Modeling (P100168)

PART VI: MDA Tasks and Clarification

For each Competency, the Study Guide will 1) comment on the Learning Resources mentioned in the AAP, 2) note optional textbooks, including those described in the tasks, 3) list online textbooks, 4) list useful websites, and 5) provide links relevant to each of the objectives.

For the Tasks, the Study Guide will provide clarification, when appropriate.
PART I: Competency 203.3.1: Statistics & Probability (HS) (P100165)

The graduate understands descriptive and inferential statistics and probability from both experimental and theoretical viewpoints.

Learning Resources
Please view your AAP under Available LRs in MDA4 or MDA5 or MDC4 or MDC5 to see the most current list. Relevant courses with open enrollment include Introduction to Statistics from RIO SALADO COMMUNITY COLLEGE and Probability & Statistics from CALIFORNIA NATIONAL UNIVERSITY. Other courses include Probability and Statistics and Statistical Methods and Data Analysis from CHADRON STATE COLLEGE, but they are offered on a fixed schedule, so please check with your mentor about when they are available.

Optional Textbooks

Useful Websites
http://davidmlane.com/hyperstat/
http://vassun.vassar.edu/%7Elowry/webtext.html
http://www.psychstat.missouristate.edu/sbk00.htm
http://www.stats.gla.ac.uk/steps/glossary/index.html
http://www.stat.berkeley.edu/users/stark/SticiGui/Text/index.htm
http://en.wikipedia.org
http://www.shodor.org/interactivate/lessons/index.html#pro
http://www.sparknotes.com/math/
http://www.math.niu.edu/~rusin/known-math/index/60-XX.html
http://people.hofstra.edu/faculty/Stefan_Waner/RealWorld/finitep.html
http://www.cmu.edu/oli/index.html
http://mathdl.maa.org/mathDL/3/

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[http://vassun.vassar.edu/%7Elowry/webtext.html](http://vassun.vassar.edu/%7Elowry/webtext.html)  
[http://www.stats.gla.ac.uk/steps/glossary/index.html](http://www.stats.gla.ac.uk/steps/glossary/index.html)  
[http://www.stats.gla.ac.uk/steps/glossary/index.html](http://www.stats.gla.ac.uk/steps/glossary/index.html)  
**PERFORMANCE ASSESSMENT** |
| **4 Solve given problems using the normal, uniform, and chi-square distributions.** | [http://davidmlane.com/hyperstat/](http://davidmlane.com/hyperstat/)  
[http://vassun.vassar.edu/%7Elowry/webtext.html](http://vassun.vassar.edu/%7Elowry/webtext.html)  
[http://www.psychnet.berkeley.edu/sbk00.htm](http://www.psychnet.berkeley.edu/sbk00.htm)  
[http://www.stats.gla.ac.uk/steps/glossary/index.html](http://www.stats.gla.ac.uk/steps/glossary/index.html)  
| **5 Determine whether to accept a given null hypotheses given the results of an experiment.** | [http://davidmlane.com/hyperstat/](http://davidmlane.com/hyperstat/)  
[http://vassun.vassar.edu/%7Elowry/webtext.html](http://vassun.vassar.edu/%7Elowry/webtext.html)  
[http://www.psychnet.berkeley.edu/sbk00.htm](http://www.psychnet.berkeley.edu/sbk00.htm)  
[http://www.stats.gla.ac.uk/steps/glossary/index.html](http://www.stats.gla.ac.uk/steps/glossary/index.html)  
| **6 Describe and analyze given bivariate data using several techniques (such as scatter plots, outliers, correlation coefficients, etc.)** | [http://davidmlane.com/hyperstat/](http://davidmlane.com/hyperstat/)  
[http://vassun.vassar.edu/%7Elowry/webtext.html](http://vassun.vassar.edu/%7Elowry/webtext.html)  
[http://www.psychnet.berkeley.edu/sbk00.htm](http://www.psychnet.berkeley.edu/sbk00.htm)  
[http://www.stats.gla.ac.uk/steps/glossary/index.html](http://www.stats.gla.ac.uk/steps/glossary/index.html)  
| **7 Calculate probabilities using the axioms of probability and related theorems and concepts. (such as the addition rule, the multiplication rule, conditional probabilities, etc.)** | [http://davidmlane.com/hyperstat/](http://davidmlane.com/hyperstat/)  
[http://vassun.vassar.edu/%7Elowry/webtext.html](http://vassun.vassar.edu/%7Elowry/webtext.html)  
[http://www.stats.gla.ac.uk/steps/glossary/index.html](http://www.stats.gla.ac.uk/steps/glossary/index.html)  
| **8 State the binomial theorem and explain its role in probability and statistics.** | [http://davidmlane.com/hyperstat/](http://davidmlane.com/hyperstat/)  
[http://vassun.vassar.edu/%7Elowry/webtext.html](http://vassun.vassar.edu/%7Elowry/webtext.html)  
[http://www.stats.gla.ac.uk/steps/glossary/index.html](http://www.stats.gla.ac.uk/steps/glossary/index.html)  
**PERFORMANCE ASSESSMENT** |
PART II: Competency 203.4.1: The Calculus & Analysis (P100166)

The graduate has a firm conceptual grasp of the notions of limit, continuity, differentiability, and integration and a thorough background in the techniques of single and multi-variable calculus and their applications to other areas of mathematics and other fields.

Learning Resources
Please view your AAP under Available LRs in MDA4 or MDA5 or MDC4 or MDC5 to see the most current list. Relevant courses with open enrollment include Calculus I, Calculus II, and Calculus III from CALIFORNIA NATIONAL UNIVERSITY. Other courses include Calculus I, Calculus II, and Calculus III from CHADRON STATE COLLEGE, but they are offered on a fixed schedule, so please check with your mentor about when they are available. THINKWELL offers an independent calculus course with lecture notes, CDs, and an available website.

Optional Textbooks

Useful Websites
http://www.math.hmc.edu/calculus/tutorials/
http://www.karlscalculus.org/calculus.html
http://archives.math.utk.edu/visual.calculus/
http://www.sosmath.com/calculus/calculus.html
http://www.analyzemath.com/calculus.html
http://tutorial.math.lamar.edu/AllBrowsers/2413/2413.asp
http://mathworld.wolfram.com/topics/Calculus.html
http://www.mathematicshelpcentral.com/index.html
http://www.math.mit.edu/~djk/18_01/contents.html
http://cow.math.temple.edu/~cow/cgi-bin/manager
http://web01.shu.edu/projects/reals/index.html
http://www.cmu.edu/oli/index.html
http://mathdl.maa.org/mathDL/3/
http://www.jtaylor1142001.net/calcjat/CFrames/menu.htm

Online Textbooks
http://ocw.mit.edu/ans7870/textbooks/Strang/strangtext.htm
http://www.math.harvard.edu/~shlomo/docs/Advanced_Calculus.pdf  (58 MB!)
http://www.math.gatech.edu/~cain/notes/calculus.html

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<th>Objective</th>
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<tbody>
<tr>
<td>1 State the formal definition of limit, derivative and</td>
<td><a href="http://www.math.hmc.edu/calculus/tutorials/limits/">http://www.math.hmc.edu/calculus/tutorials/limits/</a></td>
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<td><a href="http://www.math.hmc.edu/calculus/tutorials/limit_definition/">http://www.math.hmc.edu/calculus/tutorials/limit_definition/</a></td>
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</table>
6 Examine a given function of one variable for relative extrema and points of inflection and graph the function.

7 State the Mean Value Theorem and the Fundamental Theorem of Calculus and explain their importance to the high school calculus curriculum.

8 Calculate the area under a given curve and the volume of a given solid of revolution.
<table>
<thead>
<tr>
<th>Exercise</th>
<th>Description</th>
<th>Relevant URLs</th>
</tr>
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<tbody>
<tr>
<td>9</td>
<td>Articulate how continuity and differentiability are related yet different concepts.</td>
<td><a href="http://www.math.hmc.edu/calculus/tutorials/continuity/">Link</a></td>
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<td><a href="http://www.math.hmc.edu/calculus/tutorials/">Link</a></td>
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<td></td>
<td></td>
<td><a href="http://archives.math.utk.edu/visual.calculus/">Link</a></td>
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<td><a href="http://www.analyzemath.com/calculus.html">Link</a></td>
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<td><a href="http://mathworld.wolfram.com/topics/Calculus.html">Link</a></td>
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<td></td>
<td></td>
<td><a href="http://www-math.mit.edu/~djk/18_01/contents.html">Link</a></td>
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<td>10</td>
<td>Use L’Hopital’s rule to calculate limits of indeterminate forms, (For example, what is the limit as x approaches 0 of (sinx)/x?) and explain how to determine whether L’Hopital’s rule should be used to calculate a given limit.</td>
<td><a href="http://www.math.hmc.edu/calculus/tutorials/lhopital/">Link</a></td>
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<td><a href="http://www.math.hmc.edu/calculus/tutorials/">Link</a></td>
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<td><a href="http://mathworld.wolfram.com/topics/Calculus.html">Link</a></td>
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<td><a href="http://www-math.mit.edu/~djk/18_01/contents.html">Link</a></td>
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<td><a href="http://cow.math.temple.edu/~cow/cgi-bin/manager">Link</a></td>
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<tr>
<td>11</td>
<td>Find the Taylor Series for a given elementary function (such as e^x or sinx) around the point and explain the importance of Taylor Series in calculus applications.</td>
<td><a href="http://www.math.hmc.edu/calculus/tutorials/taylors_thm/">Link</a></td>
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<td><a href="http://www.math.hmc.edu/calculus/tutorials/">Link</a></td>
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<td><a href="http://archives.math.utk.edu/visual.calculus/">Link</a></td>
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<td><a href="http://www.analyzemath.com/calculus.html">Link</a></td>
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<td><a href="http://mathworld.wolfram.com/topics/Calculus.html">Link</a></td>
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<td><a href="http://www-math.mit.edu/~djk/18_01/contents.html">Link</a></td>
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<td><a href="http://cow.math.temple.edu/~cow/cgi-bin/manager">Link</a></td>
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<td>12</td>
<td>Find the first and second partial derivatives for a given function of several variables.</td>
<td><a href="http://www.math.hmc.edu/calculus/tutorials/partialdifferentiation/">Link</a></td>
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<td><a href="http://www.math.hmc.edu/calculus/tutorials/">Link</a></td>
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<td><a href="http://archives.math.utk.edu/visual.calculus/">Link</a></td>
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13 Give examples of applications of the derivative and the definite integral to physics, economics and business, and biology that could be taught to a high school calculus class.

PERFORMANCE ASSESSMENT

14 Solve a given physics problem involving motion using calculus techniques.

PART III: Competency 203.5.1: Symbolic Logic & Set Theory (P100167)

The graduate demonstrates reasoning using the concepts, terminology and notation of symbolic logic.

Learning Resources
Please view your AAP under Available LRs in MDA4 or MDA5 or MDC4 or MDC5 to see the most current list. Relevant courses include Discrete Mathematics from CALIFORNIA NATIONAL UNIVERSITY and Discrete Mathematics from CHADRON STATE COLLEGE. The CHADRON STATE COLLEGE course is only offered on a fixed schedule, so please check with your mentor about when they are available.

Optional Textbooks
## Useful Websites
- [http://www.math.niu.edu/~rusin/known-math/index/03-XX.html](http://www.math.niu.edu/~rusin/known-math/index/03-XX.html)
- [http://people.hofstra.edu/faculty/Stefan_Waner/RealWorld/tcfinitep.html](http://people.hofstra.edu/faculty/Stefan_Waner/RealWorld/tcfinitep.html)
- [http://www.math.csusb.edu/notes/cgi/contents.cgi](http://www.math.csusb.edu/notes/cgi/contents.cgi)
- [http://library.thinkquest.org/C0126820/start.html](http://library.thinkquest.org/C0126820/start.html)
- [http://mathdl.maa.org/mathDL/3/](http://mathdl.maa.org/mathDL/3/)

## Online Textbooks
- [http://people.hofstra.edu/faculty/Stefan_Waner/RealWorld/pdfs/Logic.pdf](http://people.hofstra.edu/faculty/Stefan_Waner/RealWorld/pdfs/Logic.pdf)
- [http://www.maths.gla.ac.uk/~ajb/dvi-ps/3q-notes.pdf](http://www.maths.gla.ac.uk/~ajb/dvi-ps/3q-notes.pdf)

## Objective Links

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<th>Objective</th>
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| 1. Define the notion of countability for infinite sets and apply the definition to determine whether the natural numbers, the rational numbers and the real numbers are countable. | [http://www.cut-the-knot.org/do_you_know/countRats.shtml](http://www.cut-the-knot.org/do_you_know/countRats.shtml)  
[http://ndp.jct.ac.il/tutorials/Discrete/node83.html](http://ndp.jct.ac.il/tutorials/Discrete/node83.html)  
[http://mathworld.wolfram.com/CountableSet.html](http://mathworld.wolfram.com/CountableSet.html)  
| 2. Determine whether a given statement is true using truth tables.        | [http://mathworld.wolfram.com/TruthTable.html](http://mathworld.wolfram.com/TruthTable.html)  
[http://www.math.csusb.edu/notes/logic/lognot/lognot.html](http://www.math.csusb.edu/notes/logic/lognot/lognot.html)  
[http://people.hofstra.edu/faculty/Stefan_Waner/RealWorld/logic/logic2.html](http://people.hofstra.edu/faculty/Stefan_Waner/RealWorld/logic/logic2.html) |
| 3. Evaluate the equivalence of given logical expressions using the laws of propositions (such as De Morgan’s Laws). | [http://www.efunda.com/math/settheory/settheory.cfm](http://www.efunda.com/math/settheory/settheory.cfm)  
[http://people.hofstra.edu/faculty/Stefan_Waner/RealWorld/logic/logic2.html](http://people.hofstra.edu/faculty/Stefan_Waner/RealWorld/logic/logic2.html)  
| 4. Perform elementary operations on given sets, such as union, intersection, and complement. | [http://people.hofstra.edu/faculty/Stefan_Waner/RealWorld/Summary5.html](http://people.hofstra.edu/faculty/Stefan_Waner/RealWorld/Summary5.html)  
[http://library.thinkquest.org/C0126820/algebra.html](http://library.thinkquest.org/C0126820/algebra.html)  
[http://library.thinkquest.org/C0126820/algebra.html](http://library.thinkquest.org/C0126820/algebra.html) |
[http://library.thinkquest.org/C0126820/relations.html](http://library.thinkquest.org/C0126820/relations.html)  
[http://www.math.csusb.edu/notes/rel/rel.html](http://www.math.csusb.edu/notes/rel/rel.html) |
PART IV: Competency 203.5.2: Induction & Recursion (P100163)

The graduate understands the role of mathematical induction as a critical method of proof in mathematics and understands how recursion is used to define important relations and functions in mathematics.

Learning Resources
Please view your AAP under Available LRs in MDA4 or MDA5 or MDC4 or MDC5 to see the most current list. Relevant courses include Discrete Mathematics from CALIFORNIA NATIONAL UNIVERSITY and Discrete Mathematics from CHADRON STATE COLLEGE. The CHADRON STATE COLLEGE course is only offered on a fixed schedule, so please check with your mentor about when they are available.

Optional Textbooks (you are not required to purchase these.)

Useful Websites
http://en.wikipedia.org/wiki/Recursion
http://personal.vsnl.com/erwin/recursion.htm
http://mathworld.wolfram.com/Recursion.html
http://www.iol.ie/~jmchugh/csc302/
http://mathforum.org
http://archives.math.utk.edu/topics/
http://mathdl.maa.org/mathDL/3/

Online Textbooks
http://euclid.trentu.ca/math/sb/pcml/pcm1-16.pdf
http://www.math.upenn.edu/~wilf/gfologyLinked2.pdf

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<tr>
<td>1. Prove a given theorem using mathematical induction (such as “The sum of the first n natural numbers is n(n+1)/2).</td>
<td><a href="http://mathworld.wolfram.com/Proof.html">http://mathworld.wolfram.com/Proof.html</a> <a href="http://mathworld.wolfram.com/PrincipleofMathematicalInduction.html">http://mathworld.wolfram.com/PrincipleofMathematicalInduction.html</a> <a href="http://archives.math.utk.edu/cgi-bin/searchTopics.pl">http://archives.math.utk.edu/cgi-bin/searchTopics.pl</a> <a href="http://bigcheese.math.sc.edu/%7Esunner/numbertheory/induction/Induction.html">http://bigcheese.math.sc.edu/%7Esunner/numbertheory/induction/Induction.html</a></td>
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<td>PERFORMANCE ASSESSMENT</td>
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<tr>
<td>4. Give an example of a recursive function that might be used in the grades 9-12 mathematics curriculum and explain how it</td>
<td><a href="http://www.shodor.org/interactivate/lessons/frac1.html">http://www.shodor.org/interactivate/lessons/frac1.html</a> <a href="http://www.shodor.org/interactivate/activities/recursion/index.html">http://www.shodor.org/interactivate/activities/recursion/index.html</a></td>
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</tbody>
</table>
PART V: Competency 203.5.3: Modeling (P100168)

The graduate knows how to use mathematics concepts to investigate patterns, make generalizations, formulate mathematical models, make predictions and validate results.

Learning Resources
Please view your AAP under Available LRs in MDA4 or MDA5 or MDC4 or MDC5 to see the most current list. It is imperative that prior to starting on the tasks covering the Modeling Competency, that students have a good command of both Calculus and Statistics. Specifically, the ability to: perform linear regression on data sets, to evaluate limits, and to solve differential equations is critical. Also, students need to possess the ability to graph a scatterplot and the least squares regression line using appropriate graphing technology. The learning resources for both Statistics and Calculus have been mentioned in the listings for Parts I & II, above. Refer to those listings for relevant learning resources.

Optional Textbooks (you are not required to purchase these.)
Peck, Roxy; Olsen, Chris; Devore, Jay; Introduction to Statistics and Data Analysis, 1st edition (Pacific Grove, CA: Brooks Cole, 2001)

Useful Websites
http://people.hofstra.edu/faculty/Stefan_Waner/RealWorld/tccalcp.html
http://betterfilecabinet.com/cgi-bin/search.pl?login
http://www.math.montana.edu/frankw/ccp/modeling/discrete/snooping/learn.htm
http://www.math.montana.edu/frankw/ccp/modeling/discrete/linear/learn.htm
http://www.jtaylor1142001.net/calcjat/Solutions/ODE/SepVars/SepVars1/SepVars1Layers.htm
http://www.intmath.com/DiffEqs/2_SepV.php
http://www.ugrad.math.ubc.ca/coursedoc/math101/notes/moreApps/separable.html

Online Textbooks
http://www.math.montana.edu/frankw/ccp/modeling/topic.htm
http://www.math.dartmouth.edu/~klbooksite/

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<th>Objective</th>
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<tbody>
<tr>
<td>1 Develop a mathematical model for a given mathematical situation (such as the population of a given colony at given times).</td>
<td><a href="http://people.hofstra.edu/faculty/Stefan_Waner/RealWorld/calctopic1/regression.html">http://people.hofstra.edu/faculty/Stefan_Waner/RealWorld/calctopic1/regression.html</a></td>
</tr>
<tr>
<td>2 Test a mathematical model to determine the extent to which it accurately models the given data.</td>
<td><a href="http://people.hofstra.edu/faculty/Stefan_Waner/RealWorld/tccalcp.html">http://people.hofstra.edu/faculty/Stefan_Waner/RealWorld/tccalcp.html</a></td>
</tr>
<tr>
<td>3 After testing a mathematical model, either modify it based on testing results, or argue that it is an appropriate model.</td>
<td><a href="http://people.hofstra.edu/faculty/Stefan_Waner/RealWorld/tccalcp.html">http://people.hofstra.edu/faculty/Stefan_Waner/RealWorld/tccalcp.html</a></td>
</tr>
<tr>
<td>4 Use a mathematical model that you have tested, and if necessary, modified, to make predictions concerning the given data and to validate the results found. <strong>PERFORMANCE ASSESSMENT</strong></td>
<td><a href="http://people.hofstra.edu/faculty/Stefan_Wane">http://people.hofstra.edu/faculty/Stefan_Wane</a> r/RealWorld/tccalcp.html</td>
</tr>
<tr>
<td>5 Create alternate models to one you have previously created and compare and contrast the alternate models to the original. <strong>PERFORMANCE ASSESSMENT</strong></td>
<td><a href="http://people.hofstra.edu/faculty/Stefan_Wane">http://people.hofstra.edu/faculty/Stefan_Wane</a> r/RealWorld/tccalcp.html</td>
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### PART VI: MDA Tasks (performance assessments) and Clarification

#### 203.3.1-03 (Task 1)

Write an essay of 250-500 words that explains the relationship between statistical inference and the theory of probability and sampling.

**Essential Components of the Essay:**
- **Paragraph 1** Define statistical inference. Define the theory of probability.
- **Paragraph 2 and following:** Explain the relationship between statistical inference and the theory of probability and sampling.
  
  Give at least 2 examples to support this.

#### 203.3.1-08 (Task 2)

The Binomial Theorem plays an important role in probability and statistics. In a short essay: 1) State and describe the binomial theorem. 2) Identify three roles that the Binomial Theorem plays in probability and statistics. 3) Provide one example of how the binomial theorem is used in solving probability and statistics problems. Be sure to show or explain the steps required to solve this problem.

**Elements:** Incorporate the following components in the essay.

- **Introduction (1-2 paragraphs):** State and describe the Binomial Theorem.
- **Discussion:** Identify three roles that the Binomial Theorem plays in probability and statistics. Provide one example of how the binomial theorem is used in solving probability and statistics problems, showing or explaining steps required to solve the problem.
- **Conclusion (1 paragraph):** Summarize the role of the Binomial Theorem plays in probability and statistics.

#### 203.4.1-01 (Task 3)

**Part I:** See Figure I, Part I below.

**Part II:** Use the definition of the derivative to prove that \( D_x(mx + b) = m \).

**Part III:** State the definition of definite integral and discuss its relationship to the derivative.

**Figure 1 Part I:** Use the formal delta-epsilon definition of \( \lim_{x \to a} f(x) = L \) to prove that \( \lim_{x \to c} (mx + b) = mc + b \).

#### 203.4.1-03 (Task 4)

**Part I:** Given that \( \lim_{x \to a} f(x) = L \), prove that \( \lim_{x \to a} [x + f(x)] = a + L \). Your proof cannot assume that the limit of a sum of two functions is the sum of their individual limits. You must use the delta-epsilon definition of limit in your proof.
**Part II:** Refer to Part I to complete Part II. Your proof cannot assume that the limit of the product of two functions is the product of their individual limits. You must use the delta-epsilon definition of limit in your proof. You may use your result from Part I.

\[
\lim_{x \to a} f(x) = L \quad \text{and} \quad \lim_{x \to a} [x \cdot f(x)] = aL
\]

Given that \( x \to a \), prove that \( \lim_{x \to a} [x \cdot f(x)] = aL \).

203.4.1-07 (Task 5)

**Part I: The Mean Value Theorem**

State the Mean Value Theorem. Use it to explain how a policeman who knows that a driver's average rate of travel was over the speed limit, can prove that the driver was in fact speeding at some point in time.

**Part II: The Fundamental Theorem of Calculus**

A. One part of the Fundamental Theorem of Calculus may be stated as shown in Figure 1 below. This statement emphasizes the idea that the definite integral can be used to calculate accumulated rates of change. Give two sample problems complete with solutions that could be used to demonstrate this particular aspect of the Fundamental Theorem.

B. The other part of the Fundamental Theorem of Calculus may be stated as shown in Figure 2 below. One use of this statement is that we can now define functions whose independent variable functions as the endpoint of a definite integral. In fact, the natural logarithm function can be defined this way. Your task is to 1) define the natural logarithm function in terms of the area under the curve in Figure 3 below, and 2) use this definition to find the derivative of \( y = \ln x \), and 3) use the derivative formula from # 2 to prove that \( \ln(a) + \ln(b) = \ln(ab) \).

**Figure 1:** If \( f^{'} \) is continuous on \([a,b]\), then \( \int_{a}^{b} f^{'}(x) \, dx = f(b) - f(a) \).

**Figure 2:** If \( G(x) = \int_{a}^{x} f(t) \, dt \), then \( G \) is an antiderivative of \( f \).

**Figure 3:** \( y = \frac{1}{x} \)

203.4.1-13 (Task 6)

You are a calculus instructor at the local high school. You have just finished your intended syllabus for the year which includes all the topics taught in a traditional one semester college calculus class. Your task is to create two take-home assignments that will illustrate the application of the calculus to other disciplines. The first assignment should present an application of differential calculus to economics, biology, or physics. The second assignment should present an application of integral calculus to economics, biology, or physics (but you must choose a different field than the first assignment). Each assignment should consist of: 1) an introduction to the application, 2) an example using that application, and 3) a follow-up problem for the students to do (include an answer key).

203.5.2-02 (Task 7)

Create a proof by mathematical induction that the sum of the first \( n \) natural numbers is equal to \( n(n+1)/2 \).

203.5.2-05 (Task 8)

As a secondary mathematics teacher, provide an example of a recursively defined function, how it will be introduced to your students, and possible applications of the function. Elements to incorporate: An example that is representative of recursive functions. Explain why the example is appropriate for grade 9-12 students.
Describe how you would introduce the example to your students.
Describe possible applications of the function.

203.5.3-01 (Task 9)

**Please note:** In order to complete this task, you will need either 1. A TI-83 graphing calculator and a TI GraphLink that will allow you to send files to your computer OR 2. An Excel spreadsheet. With Excel, you can use Insert Chart to produce a scatter plot and Add Trendline to produce a linear regression.

[Note: We recommend the TI-82, TI-83, TI-83+, or TI-84 graphing calculators. Please note that graphing calculators possessing built-in Computer Algebra Systems (CAS) are not allowed to be used on competency exams. Here is a link to check out if you need to buy a graphing calculator: http://www.pricegrabber.com/search_getprod.php/masterid=610915/search=TI-83%252B

To purchase a TI GraphLink, (a.k.a. TI Connectivity Kit, ~$20) check out this website: http://education.ti.com/educationportal/sites/US/productDetail/us_ti_connectivity_kit.html]

Please note that the point of the first modeling task (#9) is to come up with a closed-form function that models the population of the US. Also, please note that tasks 9-13 build upon one another, and are meant to first find a function that models the population, and then test its efficacy, as well as find an alternate possibility for the function, based upon an investigation of its accuracy.

**Introduction:** Refer to the table in Figure 1 below. The data listed in the table is a set of population figures (in this case, the US population) in millions of people for every ten years beginning in 1790. Your task is to use the population data to find an acceptable mathematical model for the growth of the US population. Please be aware that this performance task is the first of five projects that will use this data. The four competencies that follow this one will all depend on the work you do in meeting this competency.

Let $P(t)$ be the function that gives the US population at time $t$.

**Part I:** Form a spreadsheet with four columns.

- **Column 1:** Enter the time $t$ in years after 1790 using intervals of 10 years, i.e., 1790 corresponds to $t = 0$, 1800 corresponds to $t = 10$, etc.
- **Column 2:** Enter the US population at time $t$.
- **Column 3:** Enter the growth rate $P'(t)$. Since you do not have a function for $P(t)$, you need to make approximations using the available data. Calculate $P'$ using the symmetric difference quotient calculated by using the formula shown in Figure 2 below.

*Note that this means you will have no entry for 1790 and 2000 in this column.*

- **Column 4:** Enter the relative growth rates using the formula in Figure 3 below.

**PART II:**

Assume the relative growth rate is a linear function of time calculated by using the formula in Figure 4 below.

Do a scatter plot of relative growth rate versus time. Next, fit a linear regression to the plotted points. You must supply: 1) a printout of the graph of the regression superimposed on the scatter plot (include a viewing window) and 2) the equation for the regression with $a$ and $b$ accurate to four significant digits.

**Scatterplots & Line of Best Fit or Regression Equations Weblinks:**

- http://www.statcan.ca/english/edu/power/ch9/scattergraphs/scatter.htm (Intro to Scatterplots)
- http://www.georgetown.edu/faculty/engler/Statistics/TI83.html (Using TI-83 or 84 for Scatterplots)
Part III:
In Part II, you found $a$ and $b$ such that (see Figure 5 below.)
Use $P_0=3.9$ as your initial condition to find the particular solution for this differential equation. Note: You may find it easier to solve in terms of the constants $a$ and $b$. Show all the steps in your solution.

Figure 1

<table>
<thead>
<tr>
<th>DATE</th>
<th>POPULATION</th>
<th>DATE</th>
<th>POPULATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1790</td>
<td>3.9</td>
<td>1900</td>
<td>76.0</td>
</tr>
<tr>
<td>1800</td>
<td>5.3</td>
<td>1910</td>
<td>92.0</td>
</tr>
<tr>
<td>1810</td>
<td>7.2</td>
<td>1920</td>
<td>105.7</td>
</tr>
<tr>
<td>1820</td>
<td>9.6</td>
<td>1930</td>
<td>122.8</td>
</tr>
<tr>
<td>1830</td>
<td>12.9</td>
<td>1940</td>
<td>131.7</td>
</tr>
<tr>
<td>1840</td>
<td>17.1</td>
<td>1950</td>
<td>151.3</td>
</tr>
<tr>
<td>1850</td>
<td>23.2</td>
<td>1960</td>
<td>179.3</td>
</tr>
<tr>
<td>1860</td>
<td>31.4</td>
<td>1970</td>
<td>203.3</td>
</tr>
<tr>
<td>1870</td>
<td>39.8</td>
<td>1980</td>
<td>226.5</td>
</tr>
<tr>
<td>1880</td>
<td>50.2</td>
<td>1990</td>
<td>248.7</td>
</tr>
<tr>
<td>1890</td>
<td>62.9</td>
<td>2000</td>
<td>281.4</td>
</tr>
</tbody>
</table>

Note: All but the year 2000 entries are listed in *The World Almanac Book of Facts, 1990*. The 281.4 million people for the year 2000 was obtained from the website [www.census.gov](http://www.census.gov).

Figure 2:
Calculate $P'(t)$ using the symmetric difference quotient, i.e. $P'(t) = \frac{P(t+10) - P(t-10)}{20}$.

Figure 3: Relative growth rate: $\left(\frac{P'(t)}{P(t)}\right)$.

Figure 4: Assume the relative growth rate is a linear function of time, i.e. $\frac{1}{P} \frac{dP}{dt} = b + at$.

Figure 5: $\frac{1}{P} \frac{dP}{dt} = b + at$.

Differential Equations Weblinks:
- [http://archives.math.utk.edu/topics/ordinaryDiffEq.html](http://archives.math.utk.edu/topics/ordinaryDiffEq.html)

203.5.3-02 (Task 10)
NOTE: This task requires you to test your results from the previous task (203.5.3-01, Task 9), above.

Part I:
In completing Part II of the previous task, you were asked to do a linear regression of relative growth rate versus time. Calculate the correlation coefficient $r$ to four significant digits for your linear regression.
Part II:
What is the long term prediction of your model? Answer this question by calculating
$\lim_{t \to \infty} P(t)$ for the function $P$ that you obtained in Part III of the previous task. Justify your answer.

203.5.3-03 (Task 11)
Part I: In the previous task, you calculated the correlation coefficient for your linear regression. In 250 words or less, 1) explain the meaning of the correlation coefficient in interpreting the results of a linear regression and 2) use the calculated correlation coefficient to argue for or against the mathematical model that you used in completing 203.5.3-01, Task 9.

Part II: In the previous Task 10, you calculated $\lim_{t \to \infty} P(t)$. In 250 words or less, use the limiting value to argue for or against the mathematical model that you used in completing Task 10.

203.5.3-04 (Task 12)
Part I: Use the equation for $P(t)$ that you obtained in 203.5.3-01, Task 9 to predict the US population in a) 2010, b) 2015 and c) 2020.

Part II: Your next task is to validate the results that you found in Part I. If your model is a good one, it should be able to accurately predict the population for the year 2000. Repeat the steps that you took to find $P(t)$ while completing 203.5.3-01, Task 9 except do not use the data for the year 2000. Use your new $P(t)$ to predict the population for the year 2000. In 250 words or less, use the results from your new $P(t)$ to defend or reject the values that you calculated in Part I. (Your solution should include the new values for the linear regression, $P(2000)$ for your new $P(t)$, and the essay).

203.5.3-05 (Task 13)
Your task in this objective is to formulate a new mathematical model for US population growth. To refresh your memory, please see Part I of 203.5.3-01, Task 9. It is not necessary to redo Part I for this objective.

Part II: Assume the relative growth rate is a linear function of population at time $t$ by using the formula in Figure 1 below.

a) Do a scatter plot of relative growth rate versus population.

Clarification: In this task, note that you are now asked to assume that population is a function of the population at time $t$, and not, as previously asked, a function of time alone.

b) Next, fit a linear regression to the plotted points. You must supply 1) a printout of the graph of the regression superimposed on the scatter plot (include a viewing window) and 2) the equation for the regression with $a$ and $b$ accurate to four significant digits.

Part III: In part II, your found $a$ and $b$ such that

$$\frac{1}{P} \frac{dP}{dt} = b + aP.$$

Use $P_0=3.9$ as your initial condition to find the particular solution for this differential equation. Note: You may find it easier to solve in terms of the constants $a$ and $b$. Show all steps in your solution.

Figure 1 Assume the relative growth rate is a linear function of population at time $t$, i.e.

$$\frac{1}{P} \frac{dP}{dt} = b + aP.$$
Partial Fraction Decomposition Weblinks:
http://www.mathematicshelpcentral.com/lecture_notes/precalculus_algebra_folder/partial_fraction_decomposition.htm
http://www.math.ucdavis.edu/~kouba/CalcTwoDIRECTORY/partialfracdirectory/PartialFrac.html