This course of study outlines the sequence of learning activities that have been designed to help you develop competence in the subject area of discrete mathematics. Your competence will be assessed for HUC1 as you complete an objective assessment at a proctored test site. This course of study may take up to six weeks to complete, depending on your educational background, work experience, and the time you are able to dedicate to your studies. Consult with your mentor if you wish to accelerate your progress through this course of study.

Introduction

This course of study is aligned to the HUC1 objective assessment. The same study materials are utilized in the HUT1 performance assessment. If you have previously completed the HUT1 assessment, then you should have already completed the required study activities found in this course of study. You may wish to review the assignments here, but you are not required to repeat these activities. If you have not yet completed the HUT1 assessment, then please proceed through this course of study in full.

Overview
Welcome to Discrete Mathematics!

Discrete Mathematics prepares you to develop the competencies that you will demonstrate in the first assessment of the upper-level material of the high school mathematics endorsement studies. As such, it includes a substantial focus on proofs as well as on somewhat more abstract ideas than the earlier assessments did--namely, set theory, symbolic logic, number theory, graph theory, induction, and recursion. Be prepared to develop yourself as a proof writer and abstract thinker. One fairly well-known discrete math problem is known as the "traveling salesman problem," in which a salesman has to visit each of a set number of cities and wishes to do so in the shortest amount of traveling time possible. The techniques needed to analyze, model, and solve such a problem will also be covered here. You have likely come in contact with many of these topics before in an introductory fashion. Here you will cover them in much greater depth.

The knowledge you develop during this course of study will not only be useful for teaching mathematics, but it will also be valuable as you as you prepare for the Praxis II exam and any additional state-mandated mathematics content exams. You have completed the middle grades mathematics endorsement studies and transitioned to the upper-level material for the high school mathematics endorsement studies. This knowledge will prove invaluable as you prepare to teach discrete mathematics at a later date.

Competencies

There are several academic competencies associated with this course of study. The topics that make up these competencies will be addressed in the weekly study schedule of this course of study, which is included below. The list here provides a good preview of precisely what you will know and be able to do at the conclusion of this course of study when you have demonstrated your competence via the objective and performance assessments. The competencies for Discrete Mathematics are broken into five areas, each of which is assessed partly via the
objective assessment and partly via the performance assessment.

**Competency: Symbolic Logic**
The graduate demonstrates reasoning using the concepts, terminology, and notation of symbolic logic.

**Competency: Number Theory**
The graduate demonstrates understanding of fundamental principles of number theory.

**Competency: Set Theory**
The graduate demonstrates understanding of important principles and applications of set theory.

**Competency: Induction and Recursion**
The graduate understands the role of mathematical induction as a critical method of proof in mathematics and uses recursion to define important relations and functions in mathematics.

**Competency: Graphs**
The graduate demonstrates knowledge of graph theory and its applications.

**Learning Resources**

**Required**


**Teaching Dispositions Statement**
Please review the [WGU Statement of Teaching Dispositions](#)

**Getting Started**

To successfully complete this course of study, you need the appropriate resources to help with your learning. You should also prepare a calendar to schedule time devoted to your studies. Share your calendar with your family and friends so they are aware of your obligations.

**Acquire Learning Resources**

Arrange to obtain the learning resources listed below so there will be no delays in your studies. These items are essential for you, as this document will guide you week-by-week in the use of these materials.

**Enroll in Rosen's Discrete Mathematics**

Enroll in the following resource:

At no cost to you, go to the "Available Learning Resources" tab on your Degree Plan and enroll in the McGraw-Hill discrete math course. This is not really a course; rather, it gives you access to the Rosen textbook as an online e-textbook or as a one-time downloadable but non-copyable file (your choice).

**Explore Rosen's Online Learning Center**

**URL:** [http://highered.mcgraw-hill.com/sites/0072880082/student_view0/index.html](http://highered.mcgraw-hill.com/sites/0072880082/student_view0/index.html)

At the freely accessible website listed above is a supplement to your *Discrete Mathematics and Its Applications* textbook. This supplement has some very useful and sometimes interactive materials. Click the link above to explore it. Some links in this course of study will point to specific resources there, but you might find other parts of it useful as well, so it is important for you to view the website and make yourself familiar with it.

**Purchase Calculator Without CAS**


You will need a calculator without CAS (computer algebra system), which can be a calculator of your choice with some restrictions. Calculators possessing built-in CAS are not allowed on the objective assessment, so you should not use such a calculator while working on the topics that will be on the assessment. Programmability is acceptable, and graphing capabilities are strongly recommended. Appropriate calculators include the TI-84+ graphing calculator or equivalent calculators of other brands. Forbidden calculators include the TI-86, TI-89, TI-92, TI-94, Voyager 200, HP-48, and HP-49. If you want to download TI screenshots to your computer for inclusion in tasks, you will need a TI Connectivity Kit, which is available at the link above.

**Obtain a Notebook**

It is recommended that you acquire a notebook and use it to take notes, enabling you to turn passive reading assignments into more active and engaging assignments. Prior to assessment, reviewing your notes is an important way to prepare.

**Define "Discrete Mathematics"**

Discrete mathematics teachers are invariably asked, "So what is discrete mathematics?" Think of a short answer to that question and jot it down in your notebook. As you progress through this course of study, revisit your definition and see if it changes.

**Accessing Performance Assessments**

1. Log in to your MyWGU Student Portal.
2. Go to the "Degree Plan" tab.
3. In the list below “Course Details,” find the assessment you are working on.
4. In the “Assessment Scheduled Date” column, click “Schedule Now.”
5. A new window will come up. If there are other considerations you would like to inform the Assessment Delivery Team about, discuss them in the “Other Considerations” box that appears and then click “Continue.” If not, simply click “Continue.”
6. A request will be sent to your mentor for approval.
7. Once your mentor has approved your request, our Assessment Delivery Team will open
the tasks required for the assessment in TaskStream. You will log in to TaskStream to receive the instructions, see the rubric, and submit your assessment for grading.

Maximize Your Time
The study of discrete mathematics covers much territory and reaches new heights of mathematical rigor. To maximize your effectiveness, set goals, make a plan for reaching them, and reward yourself often! Earning a degree is a challenging endeavor, so make a plan that is strong enough to meet that challenge. Also, remember that it is not all about knowledge and skills; to be successful in your studies, your attitude towards learning and your motivation to succeed will be critical!

Calendar Management

Use the brief outline below to help you organize when you will study what topics and how you will integrate the pre-assessments and objective assessment.

Note: With approximately 24 weeks in a term and this document covering roughly one-quarter of the credit units needed for a term, you should spend no more than six weeks preparing for and passing the objective assessment. However, those weeks need not be consecutive. Some students work on two (or even three) six-week-sized areas at once, thus allowing them 12 (or 18) weeks of study for the two areas at the same time, therefore leaving more time for the knowledge to sink in. Others prefer a single-minded focus on one area.

Discrete Mathematics Outline

Week 1: Early Preassessment (optional)
  - Symbolic Logic
Week 2: Number Theory
  - Countability
Week 3: Set Theory
  - Graph Theory
  - Basement Preassessment
Week 4: Relations
  - Recursion
  - Realistic Preassessment

Optional Early Pre-Assessment

Take the preassessment for the objective assessment (PHUC) now only if you think that you have a strong background in discrete mathematics. This course of study paces you to keep you on track for SAP for the term. However, if you already have competency and can move faster, please do so by taking the preassessment now.

If the result is a passing score, immediately schedule to take HUC1. You will not have access to your performance assessment HUT1 Task 3 until you pass HUC1, so accelerating the one will help to accelerate the other. Take the preassessment under exam conditions--no reference textbook or notes, only blank paper, a writing utensil, a non-CAS calculator, and a quiet room when you have free time matching the listed time limit. Guess on any items you are unsure about.
1. Log in to your MyWGU Student Portal.
2. Go to the "My Degree Plan" tab.
3. In the list below "Course Details," find the assessment you are working on.
4. In the "Assessment Preparation" column, click "Pre-assessment."
5. In the window that pops up, click "Click here to refer for this preassessment." A request will be sent to your mentor for approval.
6. Once your mentor has approved your request, return to the "My Degree Plan" tab and click "Pre-assessment" in the "Assessment Preparation" column.
7. In the window that pops up, click "Click here to take this pre-assessment." You will then begin the pre-assessment.

Symbolic Logic

Mathematical reasoning requires using the concepts, terminology, and notation of symbolic logic. The concepts found here are critical for your future ability to reason mathematically, so be sure to immerse yourself in these preliminaries prior to moving forward.

How well can you employ the concepts, terms, and notation of symbolic logic? How do truth tables work? Why do they work? Are there any shortcuts or rules that might make the process go more smoothly? Reflect on these sorts of questions throughout this course of study. Did you know that you can use logic to assess the validity of people’s arguments--both mathematical and in everyday language? It is true. Here you will learn just how to translate written English into mathematical symbolism and begin to analyze whether those arguments are valid.

Two major components of understanding symbolic logic are being able to construct truth tables and to determine logical equivalence.

Translating Into Logical Equivalence

Can you translate a given English sentence into its logical equivalent? What does logical equivalence mean to you? Record your thoughts in your notebook and refine them during this course of study. Consider posting your thoughts to the message board as well.

Translating Conditional Propositions Self-Assessment

URL: http://highered.mcgraw-hill.com/olcweb/cgi/pluginpop.cgi?it=swf::787::538::/sites/dl/free/0072880082/299355/ConditionalPropositions.swf::Conditional%20Propositions

Go to the link above to work through the chapter 1 self-assessment. This will give you practice in translating English statements into their logical equivalents.

Truth Tables

Constructing truth tables is a necessary skill in understanding symbolic logic. It also lays the foundation for understanding rigorous proofs. Although they can, at first glance, seem tedious and time-consuming, with practice and fluency with the rules, you will become adept at producing these quickly and accurately.

Propositional Logic Reading

Read section 1.1 of your Discrete Mathematics and Its Applications textbook and record the
major points in your notebook. Consider posting your thoughts to the message board as well.

**Additional Resource: Interactive Truth Tables**

**URL:** [http://highered.mcgraw-hill.com/sites/0072880082/student_view0/interactive_demonstration_applets.html](http://highered.mcgraw-hill.com/sites/0072880082/student_view0/interactive_demonstration_applets.html)

Go to the link above to work on truth tables via Rosen's interactive demonstration applets. These applets will generate new problems for you as well as check your work and give you feedback.

**Explore Additional Resources**

**URL:** [http://www.cs.odu.edu/~toida/nerzic/content/logic/prop_logic/truth_table/truth_table.html](http://www.cs.odu.edu/~toida/nerzic/content/logic/prop_logic/truth_table/truth_table.html)

Go to the link above to work on the interactive problems about truth tables on Dr. Toida's discrete mathematics web course.


Go to the link above to work on an interactive portion of a textbook supplement by Dr. Doug Ensley.

**URL:** [http://www.math.csusb.edu/notes/quizzes/tablequiz/tablepractice.html](http://www.math.csusb.edu/notes/quizzes/tablequiz/tablepractice.html)

Go to the link above to work on quizzes from an online course at California State University--San Bernardino.

**URL:** [http://www.math.csusb.edu/notes/cgi/tvalq1.cgi](http://www.math.csusb.edu/notes/cgi/tvalq1.cgi)

Go to the link above to work on quizzes from an online course at California State University--San Bernardino.

**URL:** [http://www.math.csusb.edu/notes/quizzes/quiz1/quiz1.html](http://www.math.csusb.edu/notes/quizzes/quiz1/quiz1.html)

Go to the link above to work on quizzes from an online course at California State University--San Bernardino.

**URL:** [http://highered.mcgraw-hill.com/sites/dl/free/0072880082/299355/ExtraExamples_1_2.pdf](http://highered.mcgraw-hill.com/sites/dl/free/0072880082/299355/ExtraExamples_1_2.pdf)

Go to the link above to work on solving a set of truth tables. The answer key is accessible with a single click.

**URL:** [http://people.hofstra.edu/faculty/Stefan_Waner/RealWorld/pdfs/Logic.pdf](http://people.hofstra.edu/faculty/Stefan_Waner/RealWorld/pdfs/Logic.pdf)
Go to the link above to access Waner and Costenoble's textbook supplement, which has very thorough explanations and connections to real life as well as some interactive exercises.

**Negation of Disjunction**

It is important to know how to find the negation of a logical statement that includes a disjunction. Here you will learn how to do just that. It will be critical for you to be able to keep this distinct from the method used to negate conjunctions.

**Propositional Equivalences Reading**

Read section 1.2 of your *Discrete Mathematics and Its Applications* textbook, focusing on De Morgan's laws. In your notebook, write down a common language interpretation of the first law and/or a mnemonic device to help you remember it.

**Self-Assessment**

**URL:** [http://highered.mcgraw-hill.com/sites/0072880082/student_view0/self_assessments.html](http://highered.mcgraw-hill.com/sites/0072880082/student_view0/self_assessments.html)

Go to the link above for an interactive quiz to check whether you understand negating propositions. It is titled "Negation of Proposition" in the alphabetical list of self-assessments.

**Negation of Conjunction**

A conjunction is a logic statement containing "and." What is the negation of an "and" statement? It is important to know how to find the negation of a logical statement that includes a conjunction. Here you will learn how to do just that. It will be critical for you to be able to keep this distinct from the method used to negate disjunctions.

**Propositional Equivalences Reading**

Read section 1.2 of your *Discrete Mathematics and Its Applications* textbook, focusing on De Morgan's laws. In your notebook, write down a common language interpretation of the first law and/or a mnemonic device to help you remember it. Consider posting your thoughts to the message board as well.

**Self-Assessment**

**URL:** [http://highered.mcgraw-hill.com/sites/0072880082/student_view0/self_assessments.html](http://highered.mcgraw-hill.com/sites/0072880082/student_view0/self_assessments.html)

Go to the link above for an interactive quiz to check whether you understand negating propositions. It is titled "Negation of Proposition" in the alphabetical list of self-assessments.

**Negation of Implication**

An implication can be thought of as an "if-then" statement. What is the negation of an "if-then" statement? It is important to know how to find the negation of a logical statement that includes an implication. Here you will learn how to do just that. It will be critical for you to be able to keep this distinct from the method used to negate biconditionals.

**Propositional Equivalences Reading**

Read section 1.2 of your *Discrete Mathematics and Its Applications* textbook, focusing on table 7. In your notebook, write down a common language interpretation for that table and/or a mnemonic device to help you remember it. Consider posting your thoughts to the message board as well.
Self-Assessment

URL: http://highered.mcgraw-hill.com/sites/0072880082/student_view0/self_assessments.html#

Go to the link above for an interactive quiz to check whether you understand negating propositions. It is titled "Negation of Proposition" in the alphabetical list of self-assessments.

Negation of Biconditional
A biconditional is an "if and only if" statement. Think of it as a two-way implication. How could you find the negation of such a statement? It is important to know how to find the negation of a logical statement that includes a biconditional. Here you will learn how to do just that. It will be critical for you to be able to keep this distinct from the method used to negate implications.

Propositional Equivalences Reading

Read section 1.2 of your Discrete Mathematics and Its Applications textbook, focusing on table 8. In your notebook, write down a common language interpretation of that table and/or a mnemonic device to help you remember it. Consider posting your thoughts to the message board as well.

Self-Assessment

URL: http://highered.mcgraw-hill.com/sites/0072880082/student_view0/self_assessments.html#

Go to the link above for an interactive quiz to check whether you understand negating propositions. It is titled "Negation of Proposition" in the alphabetical list of self-assessments.

Computing Logical Equivalence
It is useful to be able to apply the computational laws of symbolic logic in order to develop new logical equivalences instead of being forced to construct truth tables and exhaustively check every possible true-versus-false value of each variable. Do you recall how you were asked to reflect upon whether there might be some shortcuts to help with all of this? Well, here you go!

Constructing New Logical Equivalences Reading

Read section 1.2 of your Discrete Mathematics and Its Applications textbook, focusing on examples 6 and 7. In your notebook, write down a common language interpretation for any tables you find puzzling and/or create a mnemonic device to help you remember them. Consider posting your thoughts to the message board as well.

Explore Additional Resources

URL: http://highered.mcgraw-hill.com/sites/dl/free/0072880082/299355/ExtraExamples_1_2.pdf

Go to the link above for a set of logical equivalence problems to solve, with the answer key accessible with a single click.

URL: http://highered.mcgraw-hill.com/sites/dl/free/0072880082/299355/c11_interactivity.html

Go to the link above for a set of interactive problems to help you practice and give you some feedback.
Tautologies and Self-Contradictions
Translating common language into logical implications and vice versa is an important mathematical skill. In fact, this skill enables you to apply all of the theoretical knowledge to the analysis of real-world problems. It is also necessary to know when a statement must be true and when it cannot be true.

Tautologies and Self-Contradictions Reading

Read section 1.5 of your Discrete Mathematics and Its Applications textbook, focusing on the first half of it. In your notebook, write down common language interpretations for the words "tautology" and "self-contradiction." Consider posting your thoughts to the message board as well.

Additional Resource

URL: http://highered.mcgraw-hill.com/sites/dl/free/0072880082/299355/ExtraExamples_1_5.pdf

Go to the link above for a set of logical equivalence problems to solve, with the answer key accessible with a single click.

Valid Arguments and Fallacies
Translating common language into logical implications and vice versa is an important mathematical skill. It is also necessary to know when logical arguments are valid and when flaws in reasoning have appeared. Without such knowledge, it would be impossible to determine the veracity of everyday arguments.

Valid Arguments and Fallacies Reading

Read section 1.5 of your Discrete Mathematics and Its Applications textbook, focusing on the last half of it. In your notebook, write down a common language interpretation for the word "fallacy." Consider posting your thoughts to the message board as well.

Additional Resource

URL: http://highered.mcgraw-hill.com/sites/dl/free/0072880082/299355/ExtraExamples_1_5.pdf

Go to the link above for a set of logical equivalence problems to solve, with the answer key accessible with a single click.

Number Theory

Part of any good discrete mathematics course is a rigorous and sometimes abstract approach to well-known concepts, such as prime numbers or remainders when dividing integers. What do you recall about how to find the greatest common factor (GCF) or least common multiple (LCM) from elementary arithmetic? Did your experiences include finding prime factorizations? What is a prime number? What is a composite number? How do they differ? Why are primes important? Is there a biggest prime number?

Define Prime
Write your definition of a prime number in your notebook, and then complete the following activities. As you do, revise your definition. Is it correct to say that a prime number has only one
factor? Why or why not?

**Prime Numbers Reading and Exercises**

Read section 3.5 of your *Discrete Mathematics and Its Applications* textbook, refining and revising your definition of prime numbers as you read. Then complete exercises 1-6.

**Additional Resource**

**URL:** [http://mathworld.wolfram.com/PrimeNumber.html](http://mathworld.wolfram.com/PrimeNumber.html)

Go to the link above. As you read, refine and revise your definition of prime numbers. Consider posting your thoughts to the message board as well.

**Define Composite**

Write your definition of a composite number in your notebook, and then complete the following activities. As you do, revise your definition. What is a composite number? How can you break composites apart into primes? Can every composite be written as a product of primes? If so, is this factorization unique? Why or why not?

**Composite Numbers Reading and Exercises**

Read section 3.5 of your *Discrete Mathematics and Its Applications* textbook, refining and revising your definition of composite numbers as you read. Then complete exercises 1-6.

**Additional Resource**

**URL:** [http://mathworld.wolfram.com/CompositeNumber.html](http://mathworld.wolfram.com/CompositeNumber.html)

Go to the link above. As you read, refine and revise your definition of composite numbers. Consider posting your thoughts to the message board as well.

**Greatest Common Factor (GCF)**

"Greatest common factor" and "greatest common divisor" are interchangeable terms. Write your definition of greatest common factor (GCF) in your notebook, and then complete the following activities. As you do, revise your definition. Why are greatest common factors (GCFs) and greatest common divisors (GCDs) important? Where do they show up in the real world?

**Greatest Common Factor Reading and Exercises**

Read section 3.5 of your *Discrete Mathematics and Its Applications* textbook, refining and revising your definition of Greatest Common Factor. Then complete exercises 20-26.

**Additional Resource**

**URL:** [http://www.purplemath.com/modules/lcm_gcf.htm](http://www.purplemath.com/modules/lcm_gcf.htm)

Go to the link above to read an elementary description of GCF through creating a prime factorization and organizing it into columns. As you read, refine and revise your definition of GCF. Consider posting your thoughts to the message board as well.

**Least Common Multiple (LCM)**

Write your definition of least common multiple (LCM) in your notebook, and then complete the following activities. As you do, revise your definition. What is the mathematical definition of
LCM? Why is the LCM important? Where do you see it in day-to-day life?

**Least Common Multiple Reading and Exercises**

Read section 3.5 of your *Discrete Mathematics and Its Applications* textbook, refining and revising your definition of Least Common Multiple as you read. Then complete exercises 20-26.

**Additional Resource**

**URL:** [http://www.purplemath.com/modules/lcm_gcf.htm](http://www.purplemath.com/modules/lcm_gcf.htm)

Go to the link above to read an elementary description of LCM by creating a prime factorization and organizing it into columns. As you read, refine and revise your definition of LCM. Consider posting your thoughts to the message board as well.

**Modular Arithmetic**

Describe modular arithmetic to a friend or post your musings to the math content message board. Then write your understanding of modular arithmetic in your notebook. Note any changes to your understanding as you complete the activities below. Why is modular arithmetic also referred to as "clock arithmetic"?

**Modular Arithmetic Reading and Exercises**

Read section 3.4 of your *Discrete Mathematics and Its Applications* textbook, focusing on the subsection titled "Modular Arithmetic." Consider how modular arithmetic employs remainders. Note the difference between the behavior of positive and negative numbers under modular arithmetic. How can subtraction be compared to the addition of negative numbers under modular arithmetic? Complete exercises 11, 12, 16, 17, and 18.

**Explore Additional Resources**

**URL:** [http://pages.sbcglobal.net/george.bunson/Programming/modularArithmetic.htm](http://pages.sbcglobal.net/george.bunson/Programming/modularArithmetic.htm)

Go to the link above. Modular arithmetic is often called "clock arithmetic." This site describes modular arithmetic in terms of an everyday clock. It also includes a description of how to deal with negative numbers.

**Euclidean Algorithm**

How does the Euclidean algorithm relate to greatest common factor (GCF)? Revisit your journal entries for GCF if needed and consider how smoothly the Euclidean algorithm allows you to compute GCF. Feel your mathematical prowess grow!

**Euclidean Algorithm Reading and Exercises**

Read the subsection "The Euclidean Algorithm" of section 3.6 of your *Discrete Mathematics and Its Applications* textbook. Complete exercises 23-26. As you read and work, consider what is familiar to you and what is new. Record any questions that arise regarding the Euclidean algorithm in your notebook and see if you can answer them as you read this section. Consider posting your thoughts to the message board as well.

**Countability**

A rigorous mathematical approach to the ideas of counting and infinity are necessary...
components of any good discrete mathematics course. Why are some sets countable while others are not? Are any infinite sets countable? Reflect on these questions, and record your answers in your notebook. Revise when necessary as your understanding of this important concept develops.

**Countability and Countably Infinite**

Sets are finite, countably infinite, or uncountably infinite in size. Recognizing those three sizes is a necessary part of understanding set theory. Record your understanding of the three set sizes in your notebook. Include many examples for each of the set sizes as well. Also important are counterexamples, so be sure to challenge yourself to create sets that do not conform to the definitions.

**Cardinality Reading**

Read the subsection "Cardinality" of section 2.4 of your *Discrete Mathematics and Its Applications* textbook.

**Additional Resource**

**URL:** [http://www.math.auckland.ac.nz/class190/lectures/Dodge-Ball-handout.pdf](http://www.math.auckland.ac.nz/class190/lectures/Dodge-Ball-handout.pdf)

Learn how to play the game of mathematical dodgeball with a logically-minded friend at the link above, and play it repeatedly. When you are convinced you know a winning strategy for Player 2, consider the following issues. How would you apply the winning strategy if the game was 10 turns long? How about 100 turns? 1000? How about if the game had a countably infinite number of turns instead of a finite number?

Now reread example 21 from section 2.4 of your *Discrete Mathematics and Its Applications* textbook and think about the connection between mathematical dodgeball and Cantor's diagonalization proof that the real numbers are uncountable (see Wikipedia for a short treatment of that proof if example 21 is not enough).

**Countably Infinite Reading**

Read the subsection "Cardinality" of section 2.4 of your *Discrete Mathematics and Its Applications* textbook.

**Explore Additional Resources**


Go to the link above for a formal but readable alternative to the textbook.

**URL:** [http://www.math.unl.edu/~webnotes/classes/classAppA/classAppA.htm](http://www.math.unl.edu/~webnotes/classes/classAppA/classAppA.htm)

Go to the link above to read Dr. John L. Orr's Analysis WebNotes, which include theorems and proofs about countability that will provide a bit more rigor than the textbook.

**Set Theory**

Mathematical reasoning requires using the concepts, terminology, and notation of set theory.
Here you begin to pull all of the prior theory together and use it to solve problems involving sets and set theory. One major component of understanding set theory is being able to do set operations, but understanding the notations and definitions of set theory is also necessary. What is your comfort level with performing set operations? How well do you understand the notations and definitions of set theory? Why is set theory important? Where do you see sets in the real world? When might you be called upon to manipulate sets and their members?

**Simplify Set Operations**

Set operations follow the same rules as logical equivalencies, but the notations for set theory and symbolic logic are different. First learn the notation of sets, then figure out the correspondences with logic, and finally apply logical equivalence rules to the simplification of set expressions. To simplify set expressions, you will use the laws of the algebra of set theory to move all complement signs to apply to individual sets, with no complement signs remaining outside the parentheses.

**Sets Reading and Exercises**

Read section 2.1 of your *Discrete Mathematics and Its Applications* textbook and complete exercises 1-9. Record your responses and your reflections on the readings in your notebook. Consider posting your thoughts to the message board as well.

**Set Operations Reading and Exercises**

Read section 2.2 of your *Discrete Mathematics and Its Applications* textbook and complete exercises 1-4, 14, 17, 19, 20, 24, 25, and 29 in your notebook. Record your reflections on the readings. Consider posting your thoughts to the message board as well.

**Explore Additional Resources**

**URL:** [http://www.cosc.brocku.ca/~duentsch/archive/methprimer1.pdf](http://www.cosc.brocku.ca/~duentsch/archive/methprimer1.pdf)

Go to the link above to read Düntsh and Gediga's *Sets, Relations, Functions* textbook, which is more advanced than your *Discrete Mathematics and Its Applications* textbook and does a very good job in chapter 1 of connecting symbolic logic and set theory. Write about that connection in your notebook, particularly how De Morgan's laws work in both systems.


Go to the link above to take an interactive quiz to check whether you understand conditional propositions.

**URL:** [http://www.cs.odu.edu/~toida/nerzic/content/questions/unit11q.html](http://www.cs.odu.edu/~toida/nerzic/content/questions/unit11q.html)

Go to the link above to read chapter 4, particularly part C, of Dr. Toida's discrete mathematics web course, which includes interactive material on sets.

**URL:** [http://www.geocities.com/basicmathsets/](http://www.geocities.com/basicmathsets/)

Go to the link above to access Martin Selditch's Interactive Basic Math Sets, which includes
interactive quizzes.


Go to the link above for a list of set problems to solve, with the answer key accessible with a single click.

**URL:** [http://highered.mcgraw-hill.com/sites/dl/free/0072880082/299355/ExtraExamples_2_2.pdf](http://highered.mcgraw-hill.com/sites/dl/free/0072880082/299355/ExtraExamples_2_2.pdf)

Go to the link above for a list of set operations problems to solve, with the answer key accessible with a single click.

**Union**

Set operations follow the same rules as logical equivalencies, but the notations for set theory and symbolic logic are different. First learn the notation of sets, then figure out the correspondences with logic, and finally apply logical equivalence rules to the simplification of set expressions. To simplify set expressions, you will use the laws of the algebra of set theory to move all complement signs to apply to individual sets, with no complement signs remaining outside the parentheses. What is the difference between the union of two sets and their intersection? Why is this difference important?

**Sets Reading and Exercises**

Read section 2.1 of your *Discrete Mathematics and Its Applications* textbook and complete exercises 1-9. Record your responses and your reflections on the readings in your notebook. Consider posting your thoughts to the message board as well.

**Set Operations Reading and Exercises**

Read section 2.2 of your *Discrete Mathematics and Its Applications* textbook and complete exercises 1-4, 14, 17, 19, 20, 24, 25, and 29 in your notebook. Record your reflections on the readings. Consider posting your thoughts to the message board as well.

**Intersection**

Set operations follow the same rules as logical equivalencies, but the notations for set theory and symbolic logic are different. First learn the notation of sets, then figure out the correspondences with logic, and finally apply logical equivalence rules to the simplification of set expressions. To simplify set expressions, you will use the laws of the algebra of set theory to move all complement signs to apply to individual sets, with no complement signs remaining outside the parentheses. What is the difference between the union of two sets and their intersection?

**Sets Reading and Exercises**

Read section 2.1 of your *Discrete Mathematics and Its Applications* textbook and complete exercises 1-9. Record your responses and your reflections on the readings in your notebook. Consider posting your thoughts to the message board as well.

**Set Operations Reading and Exercises**

Read section 2.2 of your *Discrete Mathematics and Its Applications* textbook and complete
exercises 1-4, 14, 17, 19, 20, 24, 25, and 29 in your notebook. Record your reflections on the readings as well. Others have likely posted to the message board, so if you need additional insights, check out what others have written there on these topics.

**Complement**

Set operations follow the same rules as logical equivalencies, but the notations for set theory and symbolic logic are different. First learn the notation of sets, then figure out the correspondences with logic, and finally apply logical equivalence rules to the simplification of set expressions. To simplify set expressions, you will use the laws of the algebra of set theory to move all complement signs to apply to individual sets, with no complement signs remaining outside of parentheses.

**Sets Reading and Exercises**

Read section 2.1 of your *Discrete Mathematics and Its Applications* textbook and complete exercises 1-9. Record your responses and your reflections on the readings in your notebook. Consider posting your thoughts to the message board as well.

**Set Operations Reading and Exercises**

Read section 2.2 of your *Discrete Mathematics and Its Applications* textbook and complete exercises 1-4, 14, 17, 19, 20, 24, 25, and 29 in your notebook. Record your reflections on the readings. Consider posting your thoughts to the message board as well.

**Venn Diagrams**

Draw a Venn diagram, and then complete the following activities. Note if your ability to describe data with Venn diagrams changes.

**Sets Reading and Exercises**

Read section 2.1 of your *Discrete Mathematics and Its Applications* textbook and complete exercises 10-14. Record your responses and your reflections on the readings in your notebook. Consider posting your thoughts to the message board as well.

**Set Operations Reading and Exercises**

Read section 2.2 of your *Discrete Mathematics and Its Applications* textbook and complete exercises 26-28 in your notebook. Reflect on how Venn diagrams create spatial representations of sets.

**Explore Additional Resources**

**URL:** [http://www.stat.tamu.edu/~west/applets/Venn1.html](http://www.stat.tamu.edu/~west/applets/Venn1.html)

Go to the link above to access Dr. R. Webster West's Conditional Probability Applet, which provides a visual way of conceiving sets and the operations on them.

**URL:** [http://www.math.csusb.edu/notes/quizzes/venn1/venn1.html](http://www.math.csusb.edu/notes/quizzes/venn1/venn1.html)

Go to the link above to access the Venn diagram quiz at the California State University-San Bernardino website to better visualize set operations.
Graph Theory

Graph theory includes the study of finite, complete, wheel, cycle, bipartite, and isomorphic graphs. It is important to understand the degree of vertices and paths within graphs. Euler circuits and Hamiltonian circuits are other topics under study in graph theory. Here you begin to set the stage for the eventual development of a solution to the traveling salesman problem referenced earlier.

How familiar are you with graph theory? Detail your understanding of the various types of graphs and circuits. It will be interesting to look back and see how your understanding may evolve, so be sure to record your reflections in your notebook. Consider posting your thoughts to the message board as well.

Finite Graphs

How familiar are you with graph theory? Detail your understanding of finite graphs in your notebook. What are some of the various types of finite graphs? What characteristics do they share, and how are they different?

Degree of Vertices

Record your prior knowledge of the degree of vertices in your notebook. Revisit this notebook entry when you conclude the activities below in order to fill in details and clarify your understanding. What is meant by the degree of a vertex? What insight can a vertex's degree impart?

Complete Graphs

Record your prior knowledge of complete graphs in your notebook. Revisit this notebook entry when you conclude the activities below in order to fill in details and clarify your understanding. What is a complete graph? Can you come up with a few examples? Can you produce a counterexample (or more than one)? Consider posting your thoughts to the message board as well.
Read section 9.1 of your *Discrete Mathematics and Its Applications* textbook.
Additional Resource


Go to the link above for a list of graphs problems to solve, with the answer key accessible with a single click.

**Wheel Graphs**

Record your prior knowledge of wheel graphs in your notebook. Revisit this notebook entry when you conclude the activities below in order to fill in details and clarify your understanding. What are the characteristics of wheel graphs? When are they used? How are they useful?

**Reading**

Read section 9.2 of your *Discrete Mathematics and Its Applications* textbook.

**Cycle Graphs**

Record your prior knowledge of cycle graphs in your notebook. Revisit this notebook entry when you conclude the activities below in order to fill in details and clarify your understanding. What are the characteristics of cycle graphs? When are they used? How are they useful?

**Reading**

Read section 9.2 of your *Discrete Mathematics and Its Applications* textbook.

**Bipartite Graphs**

Record your prior knowledge of bipartite graphs in your notebook. Revisit this notebook entry when you conclude the activities below in order to fill in details and clarify your understanding. What are the characteristics of bipartite graphs? When are they used? How are they useful?

**Reading**

Read section 9.2 of your *Discrete Mathematics and Its Applications* textbook.

**Isomorphic Graphs**

Record your prior knowledge of isomorphic graphs in your notebook. Revisit this notebook entry when you conclude the activities below in order to fill in details and clarify your understanding. What are the characteristics of isomorphic graphs? When are they used? How are they useful?

**Reading**

Read section 9.3 of your *Discrete Mathematics and Its Applications* textbook.

**Additional Resource**


Go to the link above for a list of isomorphic graph problems to solve, with the answer key accessible with a single click.

**Euler Circuits**

Record your prior knowledge of Euler circuits in your notebook. Revisit this notebook entry when you conclude the activities below in order to fill in details and clarify your understanding. What are the characteristics of Euler circuits? When are they used? How are they useful?
Reading

Read section 9.5 of your *Discrete Mathematics and Its Applications* textbook.

Additional Resource


Go to the link above for a list of Euler and Hamilton paths problems to solve, with the answer key accessible with a single click.

**Hamiltonian Circuits**

Record your prior knowledge of Hamiltonian circuits in your notebook. Revisit this notebook entry when you conclude the activities below in order to fill in details and clarify your understanding. What are the characteristics of Hamiltonian circuits? When are they used? How are they useful?

**Reading**

Read section 9.5 of your *Discrete Mathematics and Its Applications* textbook.

**Additional Resource**


Go to the link above for a list of Euler and Hamilton paths problems to solve, with the answer key accessible with a single click.

**Paths Within Graphs**

Record your prior knowledge of paths within graphs in your notebook. Revisit this notebook entry when you conclude the activities below in order to fill in details and clarify your understanding. Why are paths within graphs important? How are they found? When are they useful?

**Reading**

Read section 9.6 of your *Discrete Mathematics and Its Applications* textbook.

**Additional Resource**


Go to the link above to access a Java applet for working on Dijkstra's shortest path algorithm. Work out the shortest path, and then watch it be created for you step-by-step in interactive mode or watch the animated version to see the path generated.

**Assessment Preparation**

It is important to use your preassessment attempts wisely.

**Pre-Assessment: 1st or 2nd Attempt**

Take the preassessment for HUC1 if you have not already done so. Do so under exam conditions, meaning no reference textbook or notes, only blank paper, a writing utensil, a
non-CAS calculator, and a quiet room when you have free time matching the listed time limit.

**Basement Preassessment**

Take the preassessment, but do not guess when you are unsure of an answer; you do not want to accidentally get a problem right and then falsely believe you do not have to study that topic. Your score will be lower ("in the basement") than if you had guessed, but your score report will direct you to all of the right portions of this document for review and practice. It will also help you understand which topics you can afford to skim or speed through in order to give you more time on the topics that are difficult for you. If you pass the preassessment, immediately schedule the assessment.

To take the preassessment:

1. Log in to your MyWGU Student Portal.
2. Go to the "My Degree Plan" tab.
3. In the list below "Course Details," find the assessment you are working on.
4. In the "Assessment Preparation" column, click "Pre-assessment."
5. In the window that pops up, click "Click here to refer for this pre-assessment." A request will be sent to your mentor for approval.
6. Once your mentor has approved your request, return to the "My Degree Plan" tab and click "Pre-assessment" in the "Assessment Preparation" column.
7. In the window that pops up, click "Click here to take this pre-assessment." You will then begin the pre-assessment.

**Relations**

Relationships between elements of a set are formalized using the mathematical structure of relations. What is a relation? How are they related to functions? Can you come up with some examples and counterexamples? Record these in your notebook. Consider posting your thoughts to the message board as well.

**Informal R-S-T**

For each of the words "reflexive," "symmetric," and "transitive," use your notebook to record what you think they mean when used as common-language, non-mathematical words. Do not use a dictionary or a mathematical reference. Base your definitions on your experiences and hunches, and express them with ordinary language.

**Formal R-S-T**

Read section 8.1 of your *Discrete Mathematics and Its Applications* textbook to example 16. Revisit your notebook and try to draw connections between the mathematical definitions you have learned and the common-language definitions you already had. Consider posting your thoughts to the message board as well.
Reading

Read section 8.4 of your *Discrete Mathematics and Its Applications* textbook.

Explore Additional Resources


Go to the link above for a list of relations problems to solve, with the answer key accessible with a single click.


Go to the link above for a list of closures of relations problems to solve, with the answer key accessible with a single click.

**Equivalence Relation**

Equivalence relations describe a special way to relate elements of a set to each other.

*Note: It is not necessary to study this topic to complete the previous topic, which includes completing RHUT task 1, but it might be helpful.*

What are the characteristics of equivalence relations? Why are they special?

**Equivalence Relations Reading**

Read section 8.5 of your *Discrete Mathematics and Its Applications* textbook.

Explore Additional Resources


Go to the link above for a list of set problems to solve, with the answer key accessible with a single click.

**URL:** [http://www.cosc.brocku.ca/~duentsch/archive/methprimer1.pdf](http://www.cosc.brocku.ca/~duentsch/archive/methprimer1.pdf)

Go to the link above to access section 2.4 of Düntsh and Gediga's *Sets, Relations, Functions* textbook, which will help you understand why an equivalence relation on a set and on a partition of a set are the same thing.

**Matrix of Relation**

Write your definition of matrix of relation, and then do the following activities and see if your definition continues to hold. If, after doing the activities, your definition needs no changes, then congratulations! If not, what is it that you need to change or refine? Does this mean that linear algebra and discrete math might be related?

**Reading**

Read the first half of section 8.3 of your *Discrete Mathematics and Its Applications* textbook and record the key points in your notebook. This shows you how to represent a relation on $n$
elements with an \( n \text{-by-} n \) matrix.

**Directed Graph of Relation**

Write your definition of directed graphs of relation, and then do the following activities and see if your definition continues to hold. Be sure to try to create some of these on your own. In what real-world phenomenon would you expect to find these sorts of structures?

**Reading**

Read the second half of section 8.3 of your *Discrete Mathematics and Its Applications* textbook. This shows you how to represent a relation on \( n \) elements with a graph with \( n \) vertices.

**Recursion**

Recursion is used to define important relations and functions in mathematics. What recursive relationships can you name? List some recursive functions that you have encountered before in mathematics. What are some of the properties of these recursive relationships and functions? Can you represent recursive relationships with mathematical functions? Why or why not?

**Evaluate Recursion**

It is necessary to understand recursion before being able to apply it the classroom. How could you describe or explain this topic to a high school class? What layman's terms might help increase student understanding?

**Recursive Reading**

Read section 4.3 of your *Discrete Mathematics and Its Applications* textbook through to example 5 (p. 297). The section is titled "Recursive Definitions and Structural Induction" and focuses on the first part of the title.

**Recursive Definitions Additional Resource**


Go to the link above to read Wikipedia's treatment of recursion.

**Exercises**

Do exercises 1-6 of section 4.3 of your *Discrete Mathematics and Its Applications* textbook. Record these problems in your notebook.

**Self-Assessment**


Go to the link above to access an interactive quiz to check whether you understand recursion.

**Translate From Recursive to Closed Form**

Many functions can be expressed both as a function (the explicit or closed-form definition) and as a recursion (the recursive or implicit definition). It is important to be able to translate between these forms. Is one direction easier than the other? Why do you think this might be so? Do you think it holds for everyone?

**Exercises**
Review section 4.3 of your *Discrete Mathematics and Its Applications* textbook and complete exercises 7, 8, 23, 24, and 25.

**Optional Reading**

Read section 4.4 of your *Discrete Mathematics and Its Applications* textbook. This presents recursion within the context of a simulated computer programming language. The important thing to focus on is how the recursive approach breaks familiar functions down into their underlying actions.

**Explore Additional Resources**

**URL:** [http://www.geocities.com/capecanaveral/lab/3550/meaning.htm](http://www.geocities.com/capecanaveral/lab/3550/meaning.htm)

Go to the link above to read Tawanda Gwena's discussion of recursion, in which she gives some examples of the closed-form and recursive definitions.


Go to the link above to read a discussion of transulating between the forms.

**URL:** [http://books.heinemann.com/shared/onlineresources/E00682/FacNotes.pdf](http://books.heinemann.com/shared/onlineresources/E00682/FacNotes.pdf)

Go to the link above to read a discussion of the parallel development of closed-form and recursive models for a simple problem.

**Translate From Closed Form to Recursive Form**

Many functions can be expressed both as a function (the explicit or closed-form definition) and as a recursion (the recursive or implicit definition). It is important to be able to translate between these forms. Now you will basically go in the other direction. Which do you find easier? Why?

**Recursion Exercises**

Review section 4.3 of your *Discrete Mathematics and Its Applications* textbook and complete exercises 7, 8, 23, 24, and 25.

**Recursive Algorithm Reading**

Read section 4.4 of your *Discrete Mathematics and Its Applications* textbook. This presents recursion within the context of a simulated computer programming language. The important thing to focus on is how the recursive approach breaks familiar functions down into their underlying actions.

**Explore Additional Resources**

**URL:** [http://www.geocities.com/capecanaveral/lab/3550/meaning.htm](http://www.geocities.com/capecanaveral/lab/3550/meaning.htm)

Go to the link above to read Tawanda Gwena's discussion of recursion, in which she gives some examples of the closed-form and recursive definitions.

Go to the link above to read a discussion of translating between the forms.

**URL:** [http://books.heinemann.com/shared/onlineresources/E00682/FacNotes.pdf](http://books.heinemann.com/shared/onlineresources/E00682/FacNotes.pdf)

Go to the link above to read a discussion of the parallel development of closed-form and recursive models for a simple problem.

**Assessment Preparation**

It is important to use your preassessment attempts wisely. Below are instructions for doing just that.

**Pre-Assessment (2nd or 3rd Attempt)**

Take the preassessment for HUC1 again. Do so under exam conditions, meaning no reference textbook or notes, only blank paper, a writing utensil, a non-CAS calculator, and a quiet room when you have free time matching the listed time limit.

**Realistic Pre-Assessment**

You have already studied the topics you missed during your basement attempt, so now is the time to guess on any items you are unsure about. If this is the first time you pass the preassessment, immediately schedule the assessment. If this is the third time you have taken the preassessment (possibly you took it during your first few days studying discrete mathematics) but you have not passed it, then you may schedule the assessment only with your mentor's permission.

*Note: If you have only just passed the preassessment, you will need to wait a minimum of two weeks before you can take the assessment because the scheduling system requires referrals to be at a minimum of two weeks in the future. You are not allowed access to HUT1 Task 3 until you have passed the exam. Therefore, you might not be able to move on to the final scheduled week in this course of study right now, since the rest of it covers topics for the tasks.*

During this time, work on a different course of study, and at the same time do a bit of reviewing to make sure your knowledge stays fresh and that you stay prepared for the exam. Be sure to review your notebook and refresh your memory of all of the problems you have worked through during this study process. If you find a problem that is unclear to you, be sure to go back to the relevant section of this course of study to review that topic area.

To take the pre-assessment:

1. Log in to your MyWGU Student Portal.
2. Go to the "My Degree Plan" tab.
3. In the list below "Course Details," find the assessment you are working on.
4. In the "Assessment Preparation" column, click "Pre-assessment."
5. In the window that pops up, click "Click here to refer for this pre-assessment." A request will be sent to your mentor for approval.
6. Once your mentor has approved your request, return to the "My Degree Plan" tab and click "Pre-assessment" in the "Assessment Preparation" column.
7. In the window that pops up, click "Click here to take this pre-assessment." You will then begin the pre-assessment.

Conclusion

Congratulations! You should be proud that you have worked through all of the subjects, topics, and activities that have gotten you to this stage. Your hard effort and diligence will be rewarded by passing the upcoming assessment. By following this document, you have completed the rigorous Discrete Mathematics course. Savor your accomplishment!

Review of Major Points

You have completed a review of the main points of discrete mathematics: set theory, logic, graph theory, number theory, induction, and recursion. Perhaps you struggled with logic or induction? Take a moment to review your notebook and be sure to update any outmoded notions you may have had. Are there additional areas you now feel compelled to revisit? Can you now see where and how the topics you have learned can be applied to the real world? In what other non-mathematical disciplines do you see these techniques being most applicable?

Transfer/Application

Your rigorous understanding of discrete mathematics will allow you to bring out the best logical-deductive thinking in secondary mathematics students. Your ability to communicate about it in writing will help you teach students. The proof-writing skills you have developed in completing this course of study will be absolutely essential in many of the courses beyond as you complete a high school mathematics endorsement.

Next Steps

If you have followed this document closely, you have already completed the objective assessment and performance assessment required to pass Discrete Mathematics and Discrete Mathematics Applications, so there are no next steps necessary with respect to WGU. However, please keep track of your learning resources, because you will need to review discrete mathematics when you are studying for the Praxis II and/or state-mandated mathematics content exam.

Accessing Pre-Assessments

Complete the following pre-assessment:

- PHUC

For directions on how to receive access to pre-assessments, see the "Accessing Pre-Assessments" page.

Accessing Objective Assessments

Complete the following objective assessment:

- HUC1

For directions on how to receive access to objective assessments, see the “Accessing Objective and Outside Vendor Assessments” page.
Feedback

WGU values your input! If you have comments, concerns, or suggestions for improvement of this course, please submit your feedback using the following form:

- [Course Feedback](#)

ADA Policy

Western Governors University recognizes and fulfills its obligations under the Americans with Disabilities Act of 1990 (ADA), the Rehabilitation Act of 1973 and similar state laws. Western Governors University is committed to provide reasonable accommodation(s) to qualified disabled learners in University programs and activities as is required by applicable law(s). ADA Support Services serves as the principal point of contact for students seeking accommodations and can be contacted at [ADASupport@wgu.edu](mailto:ADASupport@wgu.edu). Further information on WGU's ADA policy and process can be viewed in the student handbook at the following link:

- [Policies and Procedures for Students with Disabilities](#)