This course requires an objective exam. It covers 3 competencies and 4 competency units.

Introduction

Overview

Calculus II is the study of the accumulation of change in relation to the area under a curve. It covers the knowledge and skills necessary to apply integral calculus of one variable and to use appropriate technology to model and solve real-life problems. Topics include:

- antiderivatives;
- indefinite integrals;
- the substitution rule;
- Riemann sums;
- the Fundamental Theorem of Calculus;
- definite integrals;
- acceleration, velocity, position, and initial values;
- integration by parts;
- integration by trigonometric substitution;
- integration by partial fractions;
- numerical integration;
- improper integration;
- area between curves;
- volumes and surface areas of revolution;
- arc length;
- work;
- center of mass;
- separable differential equations;
- direction fields; growth and decay problems; and
- sequences.

Calculus I is a prerequisite for this course.

Getting Started

Welcome to Calculus II! A variety of learning resources have been selected to assist you in your development of course competency. Primary lessons are contained within a free Pearson textbook, and reinforcement activities are included in a collection of Thinkwell videos, concept animations from the Wolfram Demonstrations Project, and a series of diagnostics within MyMathLab. These resources are designed to identify areas in need of further practice. You are encouraged to read the *Introduction* and *Preparing for Success* sections, as they set the stage for the successful completion of the course. Check out the Pacing Guide for a suggested timeline for completing the course. Once you feel that you have mastered the content, take the pre-assessment to help you decide if you are ready to complete the Objective Assessment. Course instructors are ready to help if you are having trouble with the course material – be sure to reach out if you need assistance. We hope that you enjoy the course!

Watch the following getting started video for this course:

Note: To download this video, right-click the following link and choose "Save as...": download video.

Teaching Dispositions Statement

Please review the WGU Statement of Teaching Dispositions.

Preparing for Success

You will typically begin your study of course concepts by reading a section of the **Textbook**. Once you've grasped the fundamentals of the content, build on your understanding by interacting with the other essential resources – usually a **Thinkwell video**. After you've absorbed these materials, take the corresponding **Diagnostic**. The diagnostics are short self-assessments pooled from the **Homework** and are most relevant to the competencies you need to learn. While the Diagnostic will provide only the most relevant problems, the homework will give you more practice with similar problem types. When you do the homework or take a diagnostic, the textbook provides many features useful for self-guided learning:

- "View an Example" to see the solution to a similar problem done step-by-step
- "Help Me Solve This" to go through the solution interactively
- "Video" to see a relevant presentation, if one exists
- "Animation" to view an interactive and/or animated illustration, if one exists
- "Textbook" for a direct link to the relevant section of the textbook

There are also a few larger **"Competence in…"** diagnostics that you should take under exam-like conditions – use only a whiteboard and your calculator, and quit when the built-in timer expires.

You can use **Review Results** after submitting a diagnostic to access the resources mentioned above. You should go back and retake these competence-based diagnostics until you are regularly scoring above 80% in order to be fully prepared for the WGU Pre-assessment and Objective Exam.

Course Diagnostic Calculus II

Skim "Things to know and memorize for Calculus II" to get an idea of the scope of the course.

If you already have a decent background in Calculus, Take the "Course Diagnostic Calculus II". If you score above 70%, you can probably skip to "Prepare for Pre-assessment" and fill in your gaps instead of going through the course linearly.

Technology for Learning

If you don't already have one, acquire an appropriate calculator for use on the exam.

Learning Resources

The learning resources listed in this section are required to complete the activities in this course. For many resources, WGU has provided automatic access through the course. However, you may need to manually enroll in or independently acquire other resources. Read the full instructions provided to ensure that you have access to all of your resources in a timely manner.

Automatically Enrolled Learning Resources

You will be automatically enrolled at the activity level for the following learning resources. Simply click on the links provided in the activities to access the learning materials.

Thomas' *Calculus Early Transcendentals*, 13e, the course textbook, which you will access with a link to page 1—use the table of contents in the left margin to navigate to the recommended readings.

Pearson MyMathLab, which you will access with links to the list of all possible Diagnostics, Review Results, and Homework; you can also link to the Pearson Study Plan, which is an efficient way to study when you're preparing for the Objective Exam.

Thinkwell Videos, which you will access with a link to the table of contents.

Manually Enrolled Learning Resources

Learn about WolframAlpha and Computable Documents, which allow you to use excellent interactive demonstrations and a Computer Algebra System to help you build a conceptual understanding of the mathematics you are studying. Links to specific animations are provided in the activities, but for them to work, you'll need to download and install the Wolfram CDF Player.

Supplemental Activities

There might be times when you need more information or practice than what has been provided in the course. In addition to consulting with your Course Instructor when you need help, you can access optional and supplemental activities by using the word "supplemental" in the Course Search box. These activities can be enriching, but they are not essential for becoming competent.

Explore Cohort Offerings

Here is the link to the Calculus II cohort site:

Pacing Guide

Follow this suggested schedule to complete the course in six weeks:

Week 1

- Self-Evaluation
- Antiderivatives and Indefinite Integrals
- Area and Riemann Sums
- Definite Integrals
- Fundamental Theorem of Calculus
- Substitution Rule

Week 2

- Integration Technology
- Integration Techniques

Week 3

- Numerical Integration
- Improper Integration
- Area Between Curves

Week 4

- Volumes
- Other Applications

Week 5

- Differential Equations
- Sequences

Week 6

- Pre-assessment
- Objective Exam

Contact a Course Instructor

Your Course Instructor is an important resource for you to take advantage of as you progress through your study of Calculus. Your Course Instructor will help guide your learning, answer questions, and provide valuable information. Be sure to consult your Course Instructor frequently. Contact information for the Course Instructors is available in the Course Community on the course's Homepage.

Competencies and Objectives

This course provides guidance to help you demonstrate the following 3 competencies:

- **Competency 213.1.1: Integration**
  The graduate demonstrates a conceptual understanding of integration techniques and correctly applies them.

Objectives:

Antiderivatives and Indefinite Integrals

- Determine the antiderivative of a function using knowledge of derivatives.
- Demonstrate an understanding of why a function has a family of antiderivatives.
- Determine a particular solution to a vertical motion problem given an initial condition.

Area and Riemann Sums

- Use upper and lower sums to approximate an area.
- Express a sum using sigma notation.
- Explain integration as a limiting sum.
- Definite Integrals
- Use proper notation for expressing a definite integral.
- Explain the basic properties of definite integrals, including additivity, zero width, sum and difference, and constant multiple.
- Use properties of definite integrals, such as order of integration, to solve problems.
- Explain the Mean Value Theorem for integrals in geometric terms.
- Determine the average value of a function.

Fundamental Theorem of Calculus

- Use the Fundamental Theorem of Calculus to evaluate a definite integral.
- Demonstrate an understanding of an integral as an accumulation function.

Substitution Rule

- Distinguish between a definite integral and an indefinite integral.
- Integrate using the general power rule.
- Recognize the substitution rule as the inverse of the chain rule.
- Integrate using the substitution rule.

Integration Techniques

- Simplify the evaluation of a definite integral using properties of even and odd functions.
- Integrate using basic integration formulas.
- Integrate using integration by parts.

- Describe the general approach of integration by parts, trigonometric integration, and trigonometric substitution.
- Identify standard integration technique appropriate for an integration problem.

Numerical Integration

- Use trapezoids to approximate definite integrals represented graphically.
- Explain how approximations using numerical methods can be made as accurate as necessary.

Integration Technology

- Approximate a definite integral using a graphing calculator or other technology.
- Evaluate an indefinite integral using a computer algebra system.

Improper Integration

- Demonstrate a geometric understanding of improper integrals.
- Evaluate an improper integral that has an infinite limit of integration.

- **Competency 213.1.2: Applied Integration**
  The graduate applies integration in various ways in order to solve problems, including differential equations.

Objectives:

Area Between Curves

- Explain how to find the area between two curves.
- Use integration to find the area between two curves.

Volumes

- Explain the use of integration to compute the volume of a simple solid with known cross-section.
- Compute the volume of a solid of revolution using the disk or washer method.
- Compare the use of the disk method and the shell method.

Other Applications

- Recognize a variety of applications of definite integrals, including length of curves, center of mass, and areas of surface revolution.
- Describe how to apply the concept of partitioning quantities to new situations.

Differential Equations

- Determine if a function is a solution of a differential equation.
- Determine the particular solution of a differential equation given the general solution and initial conditions.
- Match a differential equation with its slope field.
- Solve a first order differential equation by separation of variables.
- Apply a differential equation to solve a population modeling problem.

- **Competency 213.1.3: Sequences**
  The graduate demonstrates a conceptual understanding of sequences.

Objectives:

Sequences

- Determine the nth term of a given sequence.
- Derive a formula for the nth term of a given sequence.
- Determine if a sequence converges or diverges.
- Evaluate the limit of a convergent sequence.
- Demonstrate a conceptual understanding of the Squeeze Theorem and bounded sequences.

Course Content

This section provides the content and sequence for the course. It is suggested that you review this material in the order that it is presented and contact your course instructor if you need additional assistance with completing the activities.

Antiderivatives and Indefinite Integrals

In Calculus I, you learned that the derivative of a position function gives the velocity function. If you were given a velocity function, could you find its associated position function? More generally, if the derivative of a function is known, how do you go about finding the original function? This is the process of finding an antiderivative.

Antiderivatives

Do Section 4.8 (read the Textbook, take the Diagnostic, then Review Results and optionally do the Homework for extra practice)

Watch the videos for Thinkwell 9.1.1, 9.1.2, and 9.1.3.

Examine Table 4.2 and Table 8.1.

Self-Evaluation, Competence in Integration

To check your understanding of Competency 213.1.1: Integration, take the timed Diagnostic for Competence in Integration under exam-like circumstances – use only a whiteboard and your calculator, and quit when the built-in timer expires. If you score above 70%, you may consider skimming some of the following and moving rapidly to "Area Between Curves."

Area and Riemann Sums

Think about how you would find the area of an irregular figure. You could partition the figure into shapes for which you already know the area. For example, if you didn't remember the formula to find the area of a trapezoid, you could partition a trapezoid into a rectangle and two triangles. Similarly, we will partition figures with unknown areas into shapes with areas we do know.

Finite Sums

Do Section 5.1 (read the Textbook, take the Diagnostic, then Review Results and optionally do the Homework for extra practice)

Do Section 5.2 (read the Textbook, take the Diagnostic, then Review Results and optionally do the Homework for extra practice)

Watch the video for Thinkwell 9.4.1 and 9.4.2.

Interact with Riemann Sums and A Simple Illustration. What happens to the area estimates as you increase the number of rectangles?

Definite Integrals

This section formalizes the use of Riemann Sums and uses the limiting process to define the definite integral. This section also explores the relationship between continuity and integrability and interprets the definite integral as the area of a region.

Motion

Do Section 5.3 (read the Textbook, take the Diagnostic, then Review Results and optionally do the Homework for extra practice)

Watch the following Thinkwell videos:

Thinkwell Evaluating Definite Integrals: 9.4.6.

Motion: 10.1.1, 10.1.2, and 10.1.3.

Average Value of a Function: 18.1.1.

Interact with Average Value via Integrals to get an intuitive feel why it works.

Fundamental Theorem of Calculus

This course of study has defined definite integrals as the limit of Riemann Sums. This makes sense graphically, but it can be difficult to compute. This section investigates a theorem that greatly simplifies the calculation of a definite integral and establishes a connection between integration and differentiation.

The Fundamental Theorem of Calculus

Do Section 5.4 (read the Textbook, take the Diagnostic, then Review Results and optionally do the Homework for extra practice)

Watch the video for Thinkwell 9.4.3, 9.4.4, and 9.4.5.

Interact with Fundamental Theorem of Calculus. As you move the slider to the right, what function appears below the diagram, and what is its derivative relationship with the original function?

Read "Restore the Integral to the Fundamental Theorem of Calculus.

Substitution Rule

The Fundamental Theorem of Calculus shows that antiderivatives can be used to find definite integrals. However, antiderivatives can be more difficult to find than derivatives, if they exist at all! Recall that the set of all antiderivatives of a function is called the indefinite integral. This topic explores techniques, including the important substitution rule, for finding indefinite integrals.

Integration and Illustrating Integration by Substitution

Do Section 5.5 (read the Textbook through Example 3, take the Diagnostic, then Review Results and optionally do the Homework for extra practice)

Do Sections 5.6A (read the Textbook through Example 3, take the Diagnostic, then Review Results and optionally do the Homework for extra practice)

Watch video for Thinkwell 9.2.1, 9.2.2, 9.3.1, 9.3.2, 9.3.4 and 9.3.4.

Integration Technology

In this section, you will watch a video about using hand-held technology and then take a practice quiz to make sure you know how to apply those skills correctly. There are many other computational tools available via smartphone and the Internet, but the focus here is on using graphing calculators typical in schools and permitted on your course exam.

Integration Technology

Watch videos about finding definite integrals using graphing calculators; select those that match the calculator you plan to use during the WGU Calculus II Objective Exam. Take an online practice quiz to apply those skills. As you complete the 5 items, you'll get immediate feedback with the appropriate calculator syntax.

Integration Techniques

In this section, you will explore some of the more advanced integration techniques. You will be given an overview of how to use a variety of integration methods for finding indefinite integrals of more complex functions than you have worked with before. You will also be introduced to the concept of the improper integral, which is a definite integral with an unbounded limit.

Inverse Functions (Logarithm)

Do Section 7.1 (read the Textbook, take the Diagnostic, then Review Results and optionally do the Homework for extra practice)

Watch the video for Thinkwell 15.1.1 and 15.1.2.

Go to the Cohort Page and do the practice problems from the Integration cohort. If possible, attend the live session.

Using Basic Integration Formulas

Do Section 8.1 (read the Textbook, take the Diagnostic, then Review Results and optionally do the Homework for extra practice)

Review videos as needed for Thinkwell 9.1.1, 9.1.2, 9.1.3, 9.2.1, 9.2.2, 9.3.1, 9.3.2, 9.3.3, and 9.3.4.

Re-examine Table 8.1.

Integration by Parts

Do Section 8.2 (read the Textbook, take the Diagnostic, then Review Results and optionally do the Homework for extra practice)

Watch the video for Thinkwell 16.6.1, 16.6.2, 16.6.3, 16.6.4, and 16.6.5.

Interact with Integration by Parts and Reduction Formulas for Integrals.

Integrals Involving Powers of Sine and Cosine

Do Section 8.3 (read the Textbook, take the Diagnostic, then Review Results and optionally do the Homework for extra practice)

Watch the video for Thinkwell 16.2.1, 16.2.2, and 16.2.3.

Interact with Integrating Odd Powers of Sine and Cosine by Substitution.

Introduction to Trigonometric Substitution

Do Section 8.4 (read the Textbook but skip Hyperbolic Trigonometry, take the Diagnostic, then Review Results and optionally do the Homework for extra practice)

Watch the video for Thinkwell 16.7.1, 16.7.2, and 16.7.3.

Watch selections from "Integrals: Trig Substitution 1" and "Integrals: Trig Substitution 2" as necessary.

Complete the Diagnostic for 8.4 (no hyperbolic trig), then Review Results.

Introduction to Integration by Partial Fractions

Do Section 8.5 (read the Textbook through Example 1, take the Diagnostic, then Review Results and optionally do the Homework for extra practice)

Watch the video for Thinkwell 16.4.1, 16.4.2, and 16.4.3.

Interact with Partial Fraction Decomposition.

Complete the Diagnostic for 8.5 (through Example 1), then Review Results.

Numerical Integration

You now know a large variety of integration techniques to use when antiderivatives are easily (or not so easily) found. However, antiderivatives don't exist for all functions for which you would like to determine a definite integral. In these cases, you can turn to numerical approaches to solve problems.

Numerical Integration

NOTE: You will not be expected to set up the Simpson's Rule or calculate the error.

Do Section 8.7 (read the Textbook, take the Diagnostic, then Review Results and optionally do the Homework for extra practice)

Watch the video for Thinkwell 16.9.1 and 16.9.2.

Interact with Common Methods of Estimating the Area Under a Curve and Numerical Integration Using Rectangles, the Trapezoidal Rule, or Simpson's Rule. Why do some offer more accurate estimates than others for certain functions? What is the effect of increasing *n*?

Improper Integration

Not all definite integrals have finite limits, or even bounded ranges. Integrals of this type are called improper, but some can still be calculated. We explore the use of limits to calculate definite, but improper, integrals.

Improper Integrals

Do Section 8.8 (read the Textbook, take the Diagnostic, then Review Results and optionally do the Homework for extra practice)

Watch the video for Thinkwell 17.1.1, 17.1.2, and 17.1.3.

Interact with Improper Integrals. Note how the areas under the curve from 0 to 1 and from 1 to infinity can differ significantly. What happens to the definite integral of 1/*x*^*n* as *n* increases?

Go to the Cohort Page and do the practice problems from the Techniques cohort. If possible, attend the live session.

Competency: Integration

Take the timed Diagnostic for Competence in Integration under exam-like circumstances – use only a whiteboard and your calculator, and quit when the built-in timer expires. If you score above 80%, move on; otherwise, Review Results and retake the Diagnostic. If you haven't reached 80% in several cycles, contact your Course Instructor**.**

Area Between Curves

You have seen that definite integrals can be used to find the area of a region under a curve. Using the same notion of taking the limit of Riemann sums, definite integrals can also be used to find the area of a region between two curves.

Finding the Area between Two Curves

Do Section 5.6B (read the Textbook after Example 3, take the Diagnostic, then Review Results and optionally do the Homework for extra practice)

Watch the video for Thinkwell 10.2.1, 10.2.2, 10.2.3, and 10.2.4.

Interact with Area Between Curves. What do you notice when you change the upper and lower equations while varying the number of segments included in the summation?

Self-Evaluation, Competency in Applied Integration

To check your understanding of Competency 213.1.2: Applied Integration, take the timed Diagnostic for Competence in Applied Integration under exam-like circumstances – use only a whiteboard and your calculator, and quit when the built-in timer expires. If you score above 70%, you may consider skimming some of the following and moving rapidly to "Sequences."

Volumes

You will explore a variety of applications that require the use of integration to analyze measures of length and volume. Think about these techniques as you go about your daily life. Can you find and make up your own problems from the everyday things you see around you?

Cross-Sections

Do Section 6.1 (read the Textbook, take the Diagnostic, then Review Results and optionally do the Homework for extra practice)

Watch the following videos for Thinkwell:

Cross-Sections: 18.2.1 and 18.2.2.

Disks and Washers: 18.3.1, 18.3.2, 18.3.3, 18.3.4, and 18.3.5.

Interact with Slicing a Solid of Revolution. What happens to the volume as you increase the number of slices?

Interact with Volumes Using the Disk Method. How is accuracy affected by the number of disks?

Shells

Do Section 6.2 (read the Textbook, take the Diagnostic, then Review Results and optionally do the Homework for extra practice)

Watch the video for Thinkwell 18.4.1, 18.4.2, and 18.4.3.

Interact with Volumes of Solids of Revolution: Shell Method. How should you move the slider get the solid of revolution to emerge?

Other Applications

NOTE: For these other applications, you'll need to be able to **recognize the format** of the definite integrals associated to various applications.

Arc Lengths and Functions

Do Section 6.3 (read the Textbook through Example 3 and the formula for the length of a curve in terms of x=g(y), take the Diagnostic, then Review Results and optionally do the Homework for extra practice)

Watch the video for Thinkwell 18.5.1 and 18.5.2.

Use the Arc Length Approximation demonstration to understand how more line segments give better approximations, as long as long as the length of the largest line segment tends towards zero.

Interact with Arc Length and Polygonal Approximations to choose a "mode" to find the curve you want to explore. Pressing the "arc length formula" button displays the integral to find the exact arc length. Rotate and zoom in 3D. What do you notice?

Solid Revolution

Do Section 6.4 (read the Textbook but only the definitions for surface area of revolution about the x and y axes and the two corresponding examples, take the Diagnostic, then Review Results and optionally do the Homework for extra practice)

Interact with Surface Area of a Solid of Revolution to visually understand how to find the surface area of a solid of revolution.

Work

Do Section 6.5 (read the Textbook but only the definition for work and Examples 1 through 4, take the Diagnostic, then Review Results and optionally do the Homework for extra practice)

Watch the video for Thinkwell 18.6.1, 18.6.2, and 18.6.3.

Moments and Centers of Mass

NOTE: You will not be expected to set up or compute these integrals.

Do Section 6.6 (read the Textbook through Example 2, take the Diagnostic, then Review Results and optionally do the Homework)

Watch the video for Thinkwell 18.7.1 and 18.7.2.

Interact with Center of Mass of a Polygon, move the endpoints of the triangle and investigate how the center of mass moves with respect to the changes in shape.

Go to the Cohort Page and do the practice problems from the Applications cohort. If possible, attend the live session.

Differential Equations

The topic of differential equations covers a vast area that will be introduced here. A first-order differential equation is an equation that relates the values of the function to the derivative of the function. In other words, it is an equation that includes dy/dx as part of the equation.

Differential Equations

Do Section 7.2 (read the Textbook, take the Diagnostic, then Review Results and optionally do the Homework afor extra practice)

Do Section 9.3 (read the Textbook, take the Diagnostic, then Review Results and optionally do the Homework for extra practice)

Watch the following videos for Thinkwell 20.1.1, 20.1.2, 20.1.3, 20.3.1, and 20.3.2.

Direction Fields

Do Section 9.1 (read the Textbook, take the Diagnostic, then Review Results and optionally do the Homework for extra practice)

Watch the video for Thinkwell 20.1.4.

Interact with Slope Fields to see many examples and be sure to display exact solutions and change the initial conditions.

Competency: Applied Integration

Take the timed Diagnostic for Competence in Applied Integration under exam-like circumstances – use only a whiteboard and your calculator, and quit when the built-in timer expires. If you score above 80%, move on; otherwise, Review Results and retake the Diagnostic. If you haven't reached 80% in several cycles, contact your Course Instructor**.**

Sequences

A sequence is a function whose domain is the set of positive integers. A sequence is typically represented as a list of numbers separated by commas. In this subject you will study the concept of a sequence, including finding the limit of a sequence as the input value goes to infinity.

Sequences

Do Section 10.1 (read the Textbook, take the Diagnostic, then Review Results and optionally do the Homework for extra practice)

Watch the following videos for Thinkwell:

Sequences: 19.1.1, 19.1.2, and 19.1.3.

Monotonic and Bounded: 19.2.1

Competency: Sequences

Take the timed Diagnostic for Competence in Sequences under exam-like circumstances – use only a whiteboard and your calculator, and quit when the built-in timer expires. If you score above 80%, move on; otherwise, Review Results and retake the Diagnostic. If you haven't reached 80% in several cycles, contact your Course Instructor**.**

Go to the Cohort Page and do the practice problems from the Various Topics cohort. If possible, attend the live session.

Prepare for the Pre-assessment

Do the following to prepare for the Pre-assessment.

Pre-assessment Preparation

Review "Things to know and memorize for Calculus II"

Course Diagnostic

Take the timed "Course Diagnostic Calculus II" diagnostic under exam-like circumstances – use only a whiteboard and your calculator, and quit when the built-in timer expires. If you score above 80%, move on to the Pre-assessment; otherwise, Review Results and retake the Diagnostic. If you haven't reached 80% in several cycles, contact your Course Instructor.

Final Steps

Complete the pre-assessment and the objective assessment if you have not done so already.

Take Pre-assessment

In the Assessment tab above, the coaching report will show you which material(s) you ought to re-examine. Each line of the report is an exact match for one of the topics above, and each competency in the report matches one of the competency diagnostics above. The highly-weighted topics are deep and require mastery of the earlier material, so sometimes you need to review more than what the coaching report shows. For example, it is impossible to solve a *differential equation* problem without already being able to use *integration techniques*.

Take “Did I Get This?” Practice Test

Do the online version of “Did I Get This?” under conditions that are as close to the exam as you can – in a quiet space with a whiteboard, marker, calculator, and a 2-hour time limit to do the 37 multiple choice items.

Take Objective Assessment

If you do not pass, meet with your Course Instructor.